Continuous problems: optimality, complexity, tractability

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Abstract

Since a digital computer is able to store and manipulate only with finitely many real numbers, most computational problems of continuous mathematics can only be solved approximately using partial information. A branch of computational mathematics that studies the inherent difficulty of continuous problems, for which available information is partial, noisy, and priced, is called *information-based complexity (IBC)*. IBC emerged as a branch of computational mathematics some 30 years ago as a consequence of the need to study theoretical aspects of computations related to continuous problems. Since then IBC developed in different directions, see, e.g., the monographs [1, 2, 6, 7, 8, 9, 10, 11].

In IBC study, an important role, both theoretical and practical, play problems that are defined on functions of many or even infinitely many variables. In physical or chemical applications, the number of variables can be even millions. Such problems often suffer from the *curse of dimensionality*, i.e., their ε-complexity grows exponentially fast as the number d of variables increases to ∞. How to deal with the curse or, if possible, how to vanquish the curse, is a fundamental theoretical and practical question of contemporary computational mathematics. The three volume monograph [3, 4, 5] is the present state of the art in the subject.

In this talk, we first present the main ingredients and ideas behind IBC and then show how they can be applied and what results can be obtained for the particular problem of numerical integration of scalar and multivariate functions.

References

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