Data processing and refinement

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Vision vs Hearing

- High 2D resolution
- 3D reconstruction
- Different receptors allow colour vision

- Low spatial resolution
- Reconstruction of different sounds in a mixture
2D reconstruction from silhouettes

Is it possible to reconstruct a convex object given all 2D parallel-beam silhouettes?
2D reconstruction from silhouettes

NOT UNIQUELY!!
Tomography reconstruction

CT: reconstruction from X-Ray shadows
Tomography reconstruction

- Intensity of shadows: Beer-Lambert law

\[ I = I_0 \exp\left(-\int \mu(x) dx\right) \]

- \( \mu(x) \) is the attenuation coefficient varying for different tissues

- Silhouettes are special cases with infinite attenuation
Tomography reconstruction

- Radon transform

\[
\ln \frac{I_0}{I} = \int \mu(\bar{x}) dx = p(r, \theta)
\]

- Inverse Radon transform

\[
\mu(\bar{x}) = \frac{1}{2\pi^2} \int_0^\pi \int_{-\infty}^\infty \frac{1}{x_1 \cos \theta + x_2 \sin \theta - r} \frac{\partial p(r, \theta)}{\partial r} dr d\theta
\]
Tomography reconstruction

- Problem: the iRT is discontinuous!

Filtered Back-Projection
Fourier Transform

- **Task:** find the share of a basic signal in a mixture

\[ S(t) = 1.7 \sin(2\pi t / 5) - 0.8 \cos(2\pi t / 5) + 0.6 \sin(2\pi t / 5) \]

Basic waves: \( \sin(2\pi t / 5) \) (blue); \( \cos(2\pi t / 5) \) (green); \( \sin(2\pi t / 5) \) (red)
Fourier Transform

- Trick: integrate with the required mode

\[
\int_{0}^{T} S(t)\sin\left(\frac{2\pi}{T} t\right)dt =
\]

\[
\int_{0}^{T} b_1 \sin^2\left(\frac{2\pi}{T} t\right)dt + \int_{0}^{T} a_2 \sin\left(\frac{2\pi}{T} t\right)\cos\left(\frac{2\pi}{T} 2x\right)dt + \int_{0}^{T} b_3 \sin\left(\frac{2\pi}{T} t\right)\sin\left(\frac{2\pi}{T} 5x\right)dt =
\]

\[
\frac{T}{2} b_1 + 0 + 0.
\]
Fourier Transform

- Every signal can be split in frequencies $k/T$

- Frequency resolution is determined by the time range
Fourier Transform

- Complexification: deMoivre formula

\[ \exp(ix) = \cos(x) + i \sin(x) \]

\[ a \cos(\omega x) + b \sin(\omega x) = c_+ \exp(i\omega x) + c_- \exp(-i\omega x) \]

\[ a = c_+ + c_- \quad \quad b = i(c_+ - c_-). \]
Fourier Transform

- Discretise the time domain:

\[ T \rightarrow N\Delta t \]

Integral \( dt \) \( \rightarrow \) sum \( \Delta t \)
Fourier Transform

- Discrete Fourier Transform

\[ c_k = \frac{1}{N} \sum_{n=0}^{N-1} \exp(-i2\pi kn / N)R_n \]

- Aliasing
- Redundancy
Fourier Transform
Fourier Transform

- Uncertainty
  - Bandwidth
    \[
    \frac{N - 1}{2} \cdot \frac{1}{N\Delta t} \approx \frac{1}{2\Delta t}
    \]

- Time/Frequency resolution
  \[
  \Delta t \Delta f = \frac{1}{N}
  \]
Fourier Transform

- NMR-signals:
  - exponential decay
  - noise
Data refinement

- Baseline correction
- First point extrapolation
- Adopization (weighting)
- Zero filling
Averaging

MEK - CDCl₃

CH₂

CH₃

MEK - CDCl₃

CH₂

CH₃

MEK - CDCl₃

CH₂

CH₃

MEK - CDCl₃

CH₂

CH₃
THANK YOU FOR YOUR ATTENTION!