Summersemester 2012 Übungsblatt 2 May 2, 2012

Efficient Algorithms and Datastructures II

Aufgabe 1 (10 Punkte)

Show that the dual of the dual of a Linear Program (in standard form) is the Linear Program itself.

Aufgabe 2 (10 Punkte)

Let G = (V, E) be a bipartite graph. We want to show that the following is an exact LP-relaxation (i.e., always has an integral optimal solution) for the maximum matching problem in G:

maximize
$$\sum_{e \text{ } x_e} x_e$$
 subject to
$$\sum_{e: e \text{ is incident to } v} x_e \leq 1 \quad \forall v \in V$$

$$x_e \geq 0 \qquad \forall e \in E$$

An $m \times n$ matrix is said to be totally unimodular if the determinant of every square submatrix of A is +1, -1 or 0. It is known that if A is totally unimodular and b is integral, then the vertices of the polyhedron $P = \{x | Ax \le b, x \ge 0\}$ are integral. Show that the above linear program is an exact LP-relaxation.

Aufgabe 3 (10 Punkte)

Obtain the dual (D) of the LP above and observe that it finds a fractional minimum vertex cover in G.

Aufgabe 4 (10 Punkte)

Show that the Dual Linear Program (D) above has an integral 0/1 solution. Hence show that Strong Duality implies that the cardinality of a maximum matching equals the cardinality of a minimum vertex cover in a bipartite graph.