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Problem set 5
May 18, 2015
Summer Semester 2015

Online and Approximation Algorithms

Due May 25, 2015 before class!

Exercise 1 (Modified BIT - 10 points)

Recall the BIT algorithm for the list update problem which assigns a random bit to every item before any request is served. When an item is requested, then its bit is flipped. If the bit becomes 1, then the item is moved to the front of the list. Otherwise, its position does not change. We modify the algorithm as follows. If the requested item is already in front of the list, then we do not flip its bit. Show that the modified algorithm is no longer $\frac{7}{4}$-competitive.

Exercise 2 (TIMESTAMP(0) - 10 points)

Consider the TIMESTAMP(p) algorithm presented in class for serving a sequence σ of requests which ask access to the elements of a list. For the case where $p = 0$ and any pair of different elements x and y, show necessary and sufficient conditions specifying when x precedes y in the list after the algorithm has served all requests.

Exercise 3 (Data Compression - 10 points)

Consider the alphabet $\Sigma = \{a, b, n\}$ and the string S="bananaa" obtained from Σ.

- Show the encoding and decoding procedures of S by using Linear List compression and Move-To-Front algorithm, assuming that the initial list is $\{a, b, n\}$.
- Show all the steps of both directions of the Burrows-Wheeler transformation using linear space.

Exercise 4 (Linear List Compression with Limited List Length - 10 points)

Consider the alphabet Σ with n symbols. Recall that a compression with a linear list requires maintaining a linear list of all the symbols in Σ. In order to use less space, we shorten the length of the list to $n^{1/k}$, where k is a positive integer.

- Extend the compression scheme presented in class to the new setting. *Hint: Assume that we have at our disposal a fixed-length encoding of all the symbols in Σ which can be used in the case where a symbol is not in the list.*
- Show that the encoding length increases by a factor of at most $O(k)$ due to the decrease of the list's length.