4 Modelling Issues

What do you measure?

▶ Memory requirement
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4 Modelling Issues

How do you measure?

- Implementing and testing on representative inputs
  - How do you choose your inputs?
  - May be very time-consuming.
  - Very reliable results if done correctly.
  - Results only hold for a specific machine and for a specific set of inputs.

- Theoretical analysis in a specific model of computation.
  - Gives asymptotic bounds like “this algorithm always runs in time $O(n^2)$”.
  - Typically focuses on the worst case.
  - Can give lower bounds like “any comparison-based sorting algorithm needs at least $\Omega(n \log n)$ comparisons in the worst case”.
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4 Modelling Issues

Input length
The theoretical bounds are usually given by a function $f : \mathbb{N} \rightarrow \mathbb{N}$ that maps the input length to the running time (or storage space, comparisons, multiplications, program size etc.).

The input length may e.g. be
- the size of the input (number of bits)
- the number of arguments

Example 1
Suppose $n$ numbers from the interval $\{1, \ldots, N\}$ have to be sorted. In this case we usually say that the input length is $n$ instead of e.g. $n\log N$, which would be the number of bits required to encode the input.
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How to measure performance

1. Calculate running time and storage space etc. on a simplified, idealized model of computation, e.g. Random Access Machine (RAM), Turing Machine (TM).

2. Calculate number of certain basic operations: comparisons, multiplications, harddisk accesses.

Version 1 is often easier, but focusing on one type of operation makes it more difficult to obtain meaningful results.
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Turing Machine

- Very simple model of computation.
  - Only the “current” memory location can be altered.
  - Very good model for discussing computability, or polynomial vs. exponential time.
  - Some simple problems like recognizing whether input is of the form $xx$, where $x$ is a string, have quadratic lower bound.

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Random Access Machine (RAM)

- Input tape and output tape (sequences of zeros and ones; unbounded length).
- Memory unit: infinite but countable number of registers $R[0], R[1], R[2], \ldots$.
- Registers hold integers.
- Indirect addressing.

![Diagram of RAM](image)

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![Diagram of RAM](image)

Note that in the picture on the right the tapes are one-directional, and that a READ- or WRITE-operation always advances its tape.

Ernst Mayr, Harald Räcke 25/432
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Random Access Machine (RAM)

Operations

- input operations (input tape → \( R[i] \))
  - READ \( i \)
- output operations (\( R[i] \) → output tape)
  - WRITE \( i \)
- register-register transfers
  - \( R[j] := R[i] \)
  - \( R[i] := R[j] \)
- indirect addressing
  - \( R[i] := R[R[i]] \)
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loads the content of the \( R[i] \)-th register into the \( j \)-th register
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Operations

- branching (including loops) based on comparisons
  - jump \( x \)
    jumps to position \( x \) in the program;
    sets instruction counter to \( x \);
    reads the next operation to perform from register \( R[x] \)
  - jumpz \( x \ R[i] \)
    jump to \( x \) if \( R[i] = 0 \)
    if not the instruction counter is increased by 1;
  - jumpi \( i \)
    jump to \( R[i] \) (indirect jump);
- arithmetic instructions: \(+\), \(-\), \(\times\), \(/\)
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Model of Computation

- **uniform cost model**
  Every operation takes time 1.

- **logarithmic cost model**
  The cost depends on the content of memory cells:
  - The time for a step is equal to the largest operand involved;
  - The storage space of a register is equal to the length (in bits) of the largest value ever stored in it.

Bounded word RAM model: cost is uniform but the largest value stored in a register may not exceed $2^w$, where usually $w = \log_2 n$. 
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4 Modelling Issues

Example 2

Algorithm 1 RepeatedSquaring($n$)
1: $r \leftarrow 2$
2: for $i = 1 \rightarrow n$ do
3: \hspace{1em} $r \leftarrow r^2$
4: return $r$

- running time:
- space requirement:
Example 2

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1: \( r \leftarrow 2 \);
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  - uniform model: \( n \) steps
  - logarithmic model: \( 1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1 = \Theta(2^n) \)

- space requirement:
  - uniform model: \( O(1) \)
  - logarithmic model: \( O(2^n) \)
4 Modelling Issues

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1: \( r \leftarrow 2 \);
2: for \( i = 1 \rightarrow n \) do
3: \( r \leftarrow r^2 \)
4: return \( r \)

▶ running time:
  ▶ uniform model: \( n \) steps
  ▶ logarithmic model: \( 1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1 = \Theta(2^n) \)

▶ space requirement:
  ▶ uniform model: \( \Theta(1) \)
  ▶ logarithmic model: \( \Theta(2^n) \)
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There are different types of complexity bounds:

- **best-case complexity:**
  \[C_{bc}(n) := \min\{C(x) \mid |x| = n\}\]
  Usually easy to analyze, but not very meaningful.

- **worst-case complexity:**
  \[C_{wc}(n) := \max\{C(x) \mid |x| = n\}\]
  Usually moderately easy to analyze; sometimes too pessimistic.

- **average case complexity:**
  \[C_{avg}(n) := \frac{1}{|I_n|} \sum_{|x|=n} C(x)\]
  more general: probability measure \(\mu\)
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  The average cost of data structure operations over a worst case sequence of operations.

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  The algorithm may use random bits. Expected running time (over all possible choices of random bits) for a fixed input $x$. Then take the worst-case over all $x$ with $|x| = n$. 
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