Project Selection

Project selection problem:

- Set $P$ of possible projects. Project $v$ has an associated profit $p_v$ (can be positive or negative).
- Some projects have requirements (taking course EA2 requires course EA1).
- Dependencies are modelled in a graph. Edge $(u, v)$ means “can’t do project $u$ without also doing project $v$.”
- A subset $A$ of projects is feasible if the prerequisites of every project in $A$ also belong to $A$.

Goal: Find a feasible set of projects that maximizes the profit.
Project Selection

The prerequisite graph:

- \{x, a, z\} is a feasible subset.
- \{x, a\} is infeasible.
Project Selection

MinCut formulation:

- Edges in the prerequisite graph get infinite capacity.
- Add edge \( (s, u) \) with capacity \( p_u \) for nodes \( u \) with positive profit.
- Create edge \( (v, t) \) with capacity \( -p_v \) for nodes \( v \) with negative profit.
**Theorem 2**

*A* is a mincut if *A \ {s}* is the optimal set of projects.

**Proof.**

- *A* is feasible because of capacity infinity edges.
- \( \text{cap}(A, V \setminus A) = \sum_{v \in \bar{A} : p_v > 0} p_v + \sum_{v \in A : p_v < 0} (-p_v) \)

\[= \sum_{v : p_v > 0} p_v - \sum_{v \in A} p_v \]

For the formula we define \( p_s := 0 \).

The step follows by adding \( \sum_{v \in A : p_v > 0} p_v - \sum_{v \in A : p_v > 0} p_v = 0 \).

Note that minimizing the capacity of the cut \((A, V \setminus A)\) corresponds to maximizing profits of projects in *A*. 