Definition 1
For $b \geq 2a - 1$ an $(a, b)$-tree is a search tree with the following properties

1. all leaves have the same distance to the root
2. every internal non-root vertex $v$ has at least $a$ and at most $b$ children
3. the root has degree at least 2 if the tree is non-empty
4. the internal vertices do not contain data, but only keys (external search tree)
5. there is a special dummy leaf node with key-value $\infty$
7.5 \((a, b)\)-trees

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Each internal node \(v\) with \(d(v)\) children stores \(d - 1\) keys \(k_1, \ldots, k_{d-1}\). The \(i\)-th subtree of \(v\) fulfills

\[ k_{i-1} < \text{key in } i\text{-th sub-tree} \leq k_i, \]

where we use \(k_0 = -\infty\) and \(k_d = \infty\).
7.5 \((a, b)\)-trees

Example 2
7.5 \((a, b)\)-trees

Variants

- The dummy leaf element may not exist; it only makes implementation more convenient.
- Variants in which \(b = 2a\) are commonly referred to as \(B\)-trees.
- A \(B\)-tree usually refers to the variant in which keys and data are stored at internal nodes.
- A \(B^+\) tree stores the data only at leaf nodes as in our definition. Sometimes the leaf nodes are also connected in a linear list data structure to speed up the computation of successors and predecessors.
- A \(B^*\) tree requires that a node is at least \(2/3\)-full as opposed to \(1/2\)-full (the requirement of a \(B\)-tree).
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Lemma 3

Let $T$ be an $(a, b)$-tree for $n > 0$ elements (i.e., $n + 1$ leaf nodes) and height $h$ (number of edges from root to a leaf vertex). Then

1. $2a^{h-1} \leq n + 1 \leq b^h$

2. $\log_b(n + 1) \leq h \leq 1 + \log_a\left(\frac{n+1}{2}\right)$

Proof.

If $n > 0$ the root has degree at least $2$ and all other nodes have degree at least $a$. This gives that the number of leaf nodes is at least $2a^{h-1}$.

Analogously, the degree of any node is at most $b$ and, hence, the number of leaf nodes at most $b^h$.\[\square\]
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Time: $O(b \cdot h) = O(b \cdot \log n)$, if the individual nodes are organized as linear lists.

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Search(8)

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7.5 \((a, b)\)-trees
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Insert element $x$:

- Follow the path as if searching for $\text{key}[x]$.
- If this search ends in leaf $\ell$, insert $x$ before this leaf.
- For this add $\text{key}[x]$ to the key-list of the last internal node $v$ on the path.
- If after the insert $v$ contains $b$ nodes, do Rebalance($v$).
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Insert

Rebalance($v$):

- Let $k_i$, $i = 1, \ldots, b$ denote the keys stored in $v$.
- Let $j := \lfloor \frac{b+1}{2} \rfloor$ be the middle element.
- Create two nodes $v_1$ and $v_2$. $v_1$ gets all keys $k_1, \ldots, k_{j-1}$ and $v_2$ gets keys $k_{j+1}, \ldots, k_b$.
- Both nodes get at least $\lfloor \frac{b-1}{2} \rfloor$ keys, and have therefore degree at least $\lfloor \frac{b-1}{2} \rfloor + 1 \geq a$ since $b \geq 2a - 1$.
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- The key $k_j$ is promoted to the parent of $v$. The current pointer to $v$ is altered to point to $v_1$, and a new pointer (to the right of $k_j$) in the parent is added to point to $v_2$.
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7.5 ($a, b$)-trees
Insert

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Insert

Rebalance(ν):

- Let \( k_i, i = 1, \ldots, b \) denote the keys stored in \( ν \).
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- Create two nodes \( v_1 \) and \( v_2 \). \( v_1 \) gets all keys \( k_1, \ldots, k_{j-1} \) and \( v_2 \) gets keys \( k_{j+1}, \ldots, k_b \).
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7.5 \ (a,b)$-trees

Insert

(a,b)-trees

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Insert

Insert(8)
Insert

Insert(8)

7.5 \((a, b)\)-trees
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7.5 \((a, b)\)-trees
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**Insert(7)**

7.5 \((a, b)\)-trees
Insert

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7.5 \((a, b)\)-trees
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**Insert**

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\[ \begin{align*}
&1 \\
&3 \\
&5 \\
&6 \\
&7 \\
&8 \\
&10 \\
&19 \\
&3 \\
&6 \\
&10 \\
&19 \\
&14 \\
&28 \\
&1 \\
&3 \\
&5 \\
&6 \\
&7 \\
&8 \\
&10 \\
&14 \\
&19 \\
&28 \\
&\infty
\end{align*} \]

7.5 \((a, b)\)-trees
Insert

Insert(7)

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7.5 \((a, b)\)-trees
Delete element $x$ (pointer to leaf vertex):

- Let $v$ denote the parent of $x$. If $\text{key}[x]$ is contained in $v$, remove the key from $v$, and delete the leaf vertex.

- Otherwise delete the key of the predecessor of $x$ from $v$; delete the leaf vertex; and replace the occurrence of $\text{key}[x]$ in internal nodes by the predecessor key. (Note that it appears in exactly one internal vertex).

- If now the number of keys in $v$ is below $a - 1$ perform Rebalance$′(v)$.
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Delete

Rebalance’(ν):

- If there is a neighbour of ν that has at least \( a \) keys take over the largest (if right neighbor) or smallest (if left neighbour) and the corresponding sub-tree.
- If not: merge ν with one of its neighbours.
- The merged node contains at most \((a - 2) + (a - 1) + 1\) keys, and has therefore at most \(2a - 1 \leq b\) successors.
- Then rebalance the parent.
- During this process the root may become empty. In this case the root is deleted and the height of the tree decreases.

7.5 \((a, b)\)-trees
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7.5 \((a, b)\)-trees
Delete

\textbf{Delete}(10)

7.5 $(a, b)$-trees

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Delete

Delete(10)

7.5 \((a, b)\)-trees
Delete

Delete(10)

7.5 \((a, b)\)-trees
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Delete

Delete(14)

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Delete

Delete(14)
Delete

Delete(3)

7.5 \((a, b)\)-trees
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Delete(3)
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7.5 \((a, b)\)-trees
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7.5 \((a, b)\)-trees
Delete

Delete(1)

7.5 \((a, b)\)-trees
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Delete(19)
Delete

Delete(19)
Delete

Delete(19)

7.5 \((a, b)\)-trees
Delete

Delete(19)
Delete (19)
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7.5 \((a, b)\)-trees
(2, 4)-trees and red black trees

There is a close relation between red-black trees and (2, 4)-trees:
(2, 4)-trees and red black trees

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(2, 4)-trees and red black trees

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Note that this correspondence is not unique. In particular, there are different red-black trees that correspond to the same (2, 4)-tree.
(2, 4)-trees and red black trees

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(2, 4)-trees and red black trees

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Note that this correspondence is not unique. In particular, there are different red-black trees that correspond to the same \((2, 4)\)-tree.