Technische Universität München
Fakultät für Informatik
Lehrstuhl für Theoretische Informatik
Prof. Dr. Harald Räcke
Richard Stotz, Dennis Kraft

Winter term 2016 / 2017
Assignment 1
October 24, 2016

Efficient Algorithms and Data Structures I

*Deadline: October 31, 10:15 am in the **Efficient Algorithms** mailbox*

Homework 1 (5 Points)

Consider the following simple primality testing algorithm:

Algorithm 1: SimpleIsPrime(N)

for $i = 2 \ldots N-1$ **do**
    if $N \bmod i == 0$ **then**
        return N is composite!
return N is prime!

1. Show that the worst-case running time of the algorithm in the uniform cost model is $\mathcal{O}(N)$.
2. Assume that computing $p \bmod q$ takes time $\lfloor (p/q) \log p \rfloor$ in the logarithmic cost model. Show that the worst-case running time of the algorithm in the logarithmic cost model is $\mathcal{O}(N(\log N)^2)$.
3. Argue for both models that the running time of algorithm SimpleIsPrime(N) is not polynomial in the input size.

Homework 2 (5 Points)

Let $f, g : \mathbb{N} \to \mathbb{R}^+$ be two positive monotone increasing functions.
Prove or disprove the following statements. Use precise arguments based on the definition of the Landau-notation shown in the lecture.

1. $f(n) + g(n) \in \Omega(f(n))$.
2. $o(f(n)) \cap \omega(f(n)) \neq \emptyset$.
3. $\frac{1}{\Omega(n)} \subseteq O(\frac{1}{n})$.

Homework 3 (5 Points)

The pastry chefs Aaron and Beth bake n petit fours together. Aaron handles the first steps and needs $f(n)$ minutes. Beth needs $g(n)$ minutes to finish the delicious sweets.

1. Aaron conjectures that, asymptotically, they only need to know which one of them takes longer in order to estimate the total time needed. Using the basic definition of the Θ-notation, show that Aaron is right, i.e. that $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$.

2. Upon further investigation, the two chefs find that Aaron needs asymptotically at most as much time as Beth. They conclude that they only need to know how much time Beth needs in order to estimate the total time needed.

   Using the basic definition of the $\mathcal{O}$-notation, show that this conclusion is also correct, i.e. that $f(n) \in \mathcal{O}(g(n)) \implies f(n) + g(n) \in \mathcal{O}(g(n))$.

Homework 4 (5 Points)

Show by using the basic definition of the Θ-notation, that for any real constants a and b, where $b > 0$,

$$(n + a)^b = \Theta(n^b)$$

Tutorial Exercise 1

For constants $c > 0$, $0 < \varepsilon < 1/2$ and $k > 1$, arrange the following functions of n in non-decreasing asymptotic order so that $f_i(n) \in O(f_{i+1}(n))$ for two consecutive functions in your sequence. Also indicate whether $f_i(n) \in \Theta(f_{i+1}(n))$ holds or not.

$$n^k, \sqrt{n}, 2^n, n^{1+sin(n)}, \log(n!), n^{k+\varepsilon}, n^n, n, n^k(\log n)^c, n!, 2^n, 3^n, n \log\log n, n\log(n), n^\varepsilon, \log(n)^2$$

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The advanced reader who
skips parts that appear
too elementary may miss
more than the less advanced
reader who skips parts that
appear too complex.
                 - G. Pòlya
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