Technische Universität München
Fakultät für Informatik
Lehrstuhl für Theoretische Informatik
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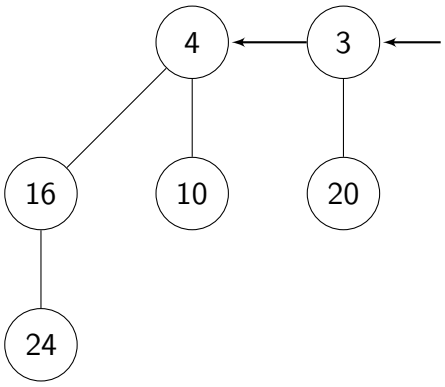
Winter term 2016 / 2017
Assignment 9
December 19, 2016

Efficient Algorithms and Data Structures I

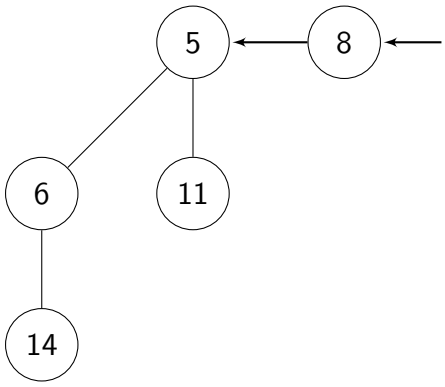
*Deadline: January 9, 2017, 10:15 am in the **Efficient Algorithms** mailbox.*

Homework 1 (5 Points)

Perform the following operations sequentially on the Binomial Heaps shown below so that they remain Binomial Heap(s) and show what Heap A looks like after each operation (always carry out each operation on the result of the previous operation):



Heap A



Heap B

(a) A.Insert(1)

(b) A.Merge(B)

(c) A.Delete-Min()

Homework 2 (5 Points)

An $m \times n$ Young tableau is an $m \times n$ matrix such that the entries of each row are in sorted order from left to right and the entries of each column are in sorted order from top to bottom. All entries are integers or ∞ (infinity), which is treated as a nonexistant element.

(a) Draw a 3×4 Young tableau containing the elements $\{4, 5, 1, 86, 25, 10, 8, 17, 20, 44\}$.

(b) Give an recursive algorithm to implement DELETE-MIN on a nonempty $m \times n$ Young tableau. Your algorithm should return the minimum entry of the tableau as well as a new tableau without this entry. The running time should be $\mathcal{O}(m + n)$.

Hint: Think about the DELETE operation of a binary heap. Try to come up with a similar recursion that recursively calls an $(m - 1) \times n$ or an $m \times (n - 1)$-subproblem. Let $p = m + n$ and solve a recurrence for the running time $T(p)$.

Homework 3 (5 Points)

n unicyclists $M_1, M_2, \ldots, M_n$ start riding their unicycles from a (straight) start line. At the start M_i and M_{i+1} are adjacent to each other. Each unicyclist M_i starts at some angle ϕ_i and keeps riding in a straight line along this direction at a constant speed $s_i > 0$. Whenever a unicyclist M_j comes across the path traversed by any other unicyclist M_i, we say that M_i defeated M_j and in that case, M_j stops riding.

(a) We call the point where M_i defeats M_j as the point of ambush $A_{i,j} \in \mathbb{R}^2$. Show that if $A_{i',j'}$ is a point of ambush which occurs closest to the start line, then i' and j' are consecutive integers.

(b) Show how to enumerate in $\mathcal{O}(n \log n)$ time all events where one unicyclist defeats another.

Homework 4 (5 Points)

We say that $f(n) \in \overset{\infty}{\Omega}(g(n))$ if there exists a positive constant c such that

$$f(n) \geq c \cdot g(n) \geq 0 \qquad \text{for infinitely many integers } n.$$

(a) Give two functions $f(n)$ and $g(n)$, such that $f(n) \in \overset{\infty}{\Omega}(g(n))$ but $f(n) \notin \Omega(g(n))$.

(b) Find inputs that cause DELETE-MIN, DECREASE-KEY, and DELETE to run in $\Omega(\log n)$ time for a binomial heap.

(c) Explain why the running times **(not necessarily worst-case)** of INSERT, MINIMUM, and MERGE are $\overset{\infty}{\Omega}(\log n)$ but not $\Omega(\log n)$ for a binomial heap.

Bonus Homework 1 (10 Bonus Points)

Note: Bonus Points improve your score for both semester halves!
True or False? Give a short explanation for each answer. If no explanation is given, then no points are awarded.

(a) $2^n \in \Theta(4^n)$.

(b) A red-black tree with n internal nodes can be stored in $\mathcal{O}(n)$ space, assuming that each individual node needs constant space.

(c) If we insert a node in a red-black tree and then immediately delete the same node, the resulting binary tree (without colours assigned to nodes) is the same as the original binary tree.

(d) Every node in an (a, b)-tree has at least a children.

(e) Insertion in a skip list of size n always takes time $\mathcal{O}(\log n)$.

Tutorial Exercise 1

For any positive integer n, show a sequence of Fibonacci heap operations that creates a Fibonacci heap consisting of just one tree that is a linear chain of n nodes.

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We wish you a merry christmas
and a happy new year!
      - The teaching assistants of EA
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