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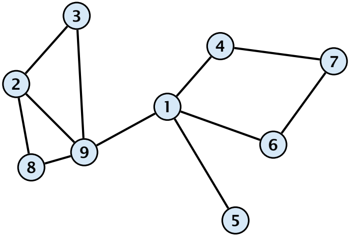

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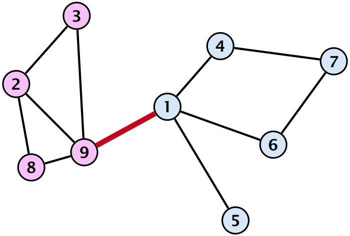

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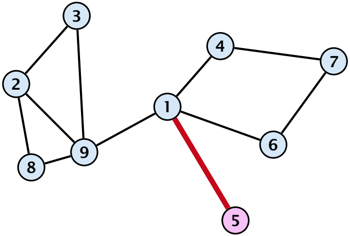

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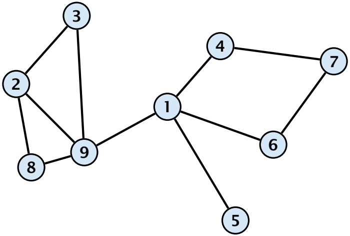

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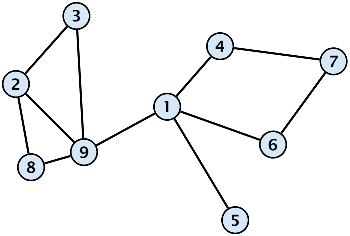

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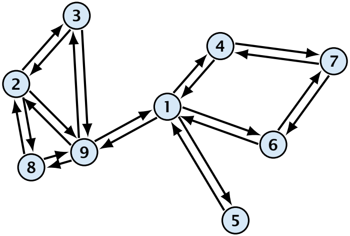

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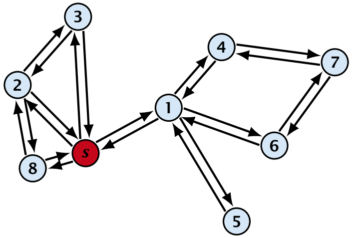

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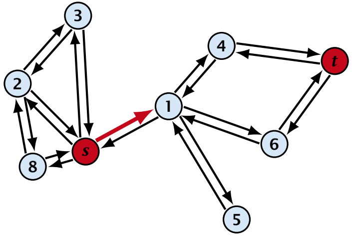

Edge Contractions

- Given a graph $G = (V, E)$ and an edge $e = \{u, v\}$.
- The graph G/e is obtained by "identifying" u and v to form a new node.
- Resulting parallel edges are replaced by a single edge, whose capacity equals the sum of capacities of the parallel edges.

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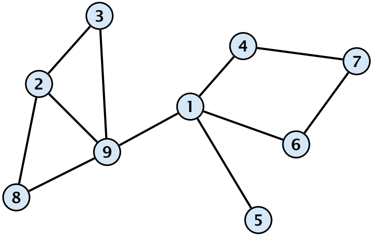

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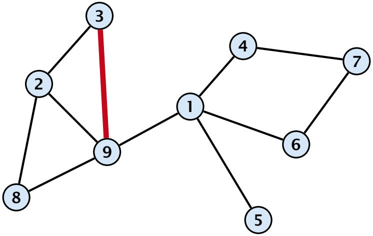

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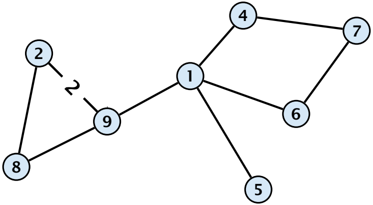

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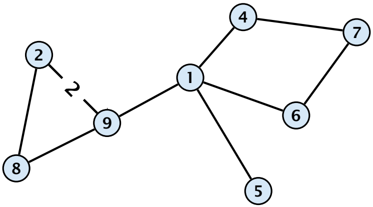

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Edge Contractions

We can perform an edge-contraction in time $\mathcal{O}(n)$.

Randomized Mincut Algorithm

Algorithm 7 KargerMincut($G = (V, E, c)$)

for $i = 1 \to n - 2$ **do**
  choose $e \in E$ randomly with probability $c(e)/c(E)$
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- What is the probability that this algorithm returns a mincut?

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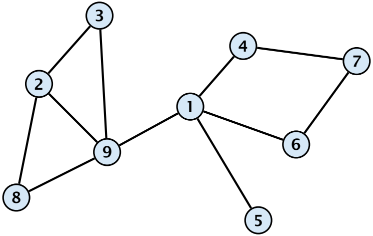

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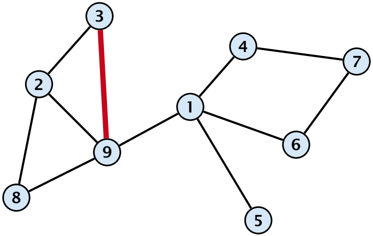

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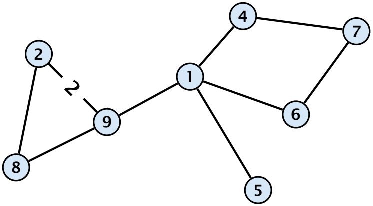

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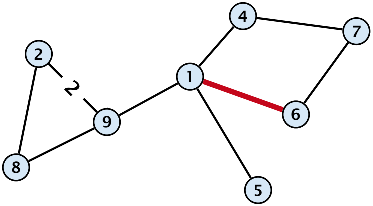

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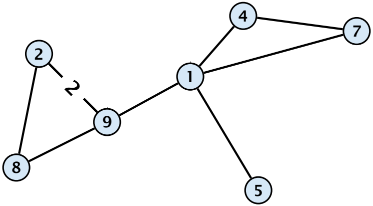

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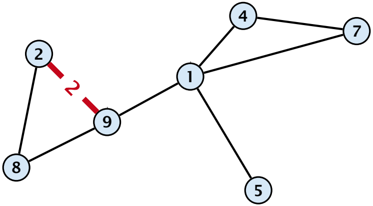

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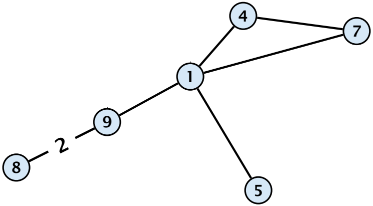

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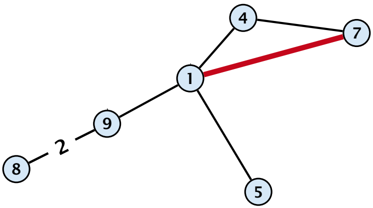

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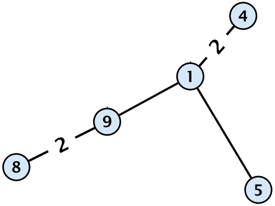

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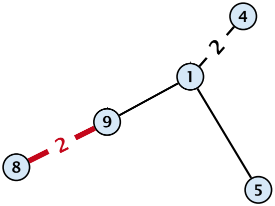

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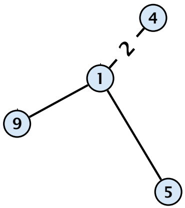

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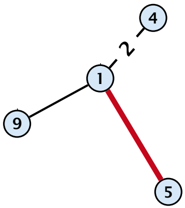

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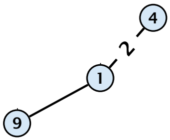

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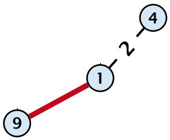

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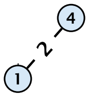

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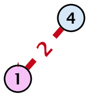

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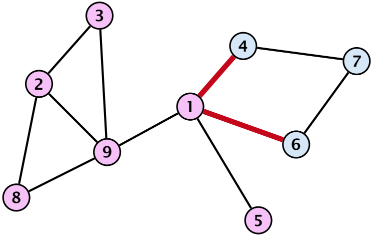

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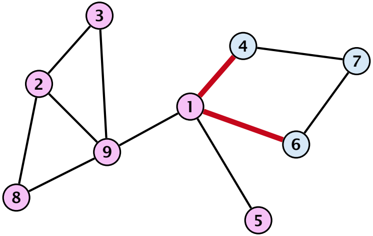

What is the probability that this algorithm returns a mincut?

Analysis

What is the probability that a given mincut A is still possible after round i?

- It is still possible to obtain cut A in the end if so far no edge in $(A, V \setminus A)$ has been contracted.

Analysis

What is the probability that we select an edge from A in iteration i?

- Let $\min = \text{cap}(A, V \setminus A)$ denote the capacity of a mincut.
- Let $\text{cap}(v)$ be capacity of edges incident to vertex $v \in V_{n-i+1}$.
- Clearly, $\text{cap}(v) \geq \min$.
- Summing $\text{cap}(v)$ over all edges gives

$$2c(E) = 2\sum_{e \in E} c(e) = \sum_{v \in V} \text{cap}(v) \geq (n - i + 1) \cdot \min$$

- Hence, the probability of choosing an edge from the cut is at most $\min/c(E) \leq 2/(n-i+1)$.

$n - i + 1$ is the number of nodes in graph $G_{n-i+1} = (V_{n-i+1}, E_{n-i+1})$, the graph at the start of iteration i.

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$$\left(1-\frac{1}{\binom{n}{2}}\right)^{\binom{n}{2}c\ln n} \leq \left(e^{-1/\binom{n}{2}}\right)^{\binom{n}{2}c\ln n} \leq n^{-c} ,$$

where we used $1-x\leq e^{-x}$.

Theorem 2
The randomized mincut algorithm computes an optimal cut with high probability. The total running time is $\mathcal{O}(n^4\log n)$.

Improved Algorithm

Algorithm 8 RecursiveMincut($G = (V, E, c)$)

for $i = 1 \rightarrow n - n/\sqrt{2}$ **do**
    choose $e \in E$ randomly with probability $c(e)/c(E)$
    $G \leftarrow G/e$
if $|V| = 2$ **return** cut-value;
$cuta \leftarrow$ RecursiveMincut(G);
$cutb \leftarrow$ RecursiveMincut(G);
return $\min\{cuta, cutb\}$

Running time:

Note that the above implementation only works for very special values of n.

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- $T(n) = 2T\left(\frac{n}{\sqrt{2}}\right) + \mathcal{O}(n^2)$
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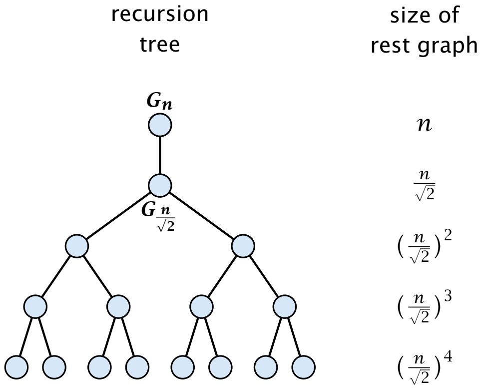
Probability of Success

The probability of contracting an edge from the mincut during one iteration through the for-loop is only

$$\frac{t(t-1)}{n(n-1)} \le \frac{t^2}{n^2} = \frac{1}{2} \ ,$$

as $t = \frac{n}{\sqrt{2}}$.

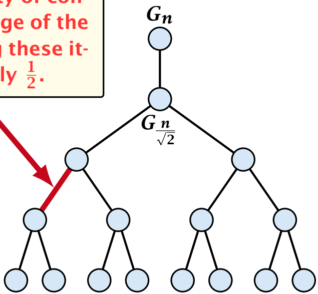
Probability of Success



We can estimate the success probability by using the following game on the recursion tree. Delete every edge with probability $\frac{1}{2}$. If in the end you have a path from the root to at least one leaf node you are successful.

Probability of Success

The probability of contracting an edge of the mincut during these iterations is only $\frac{1}{2}$.

recursion tree	size of rest graph
	n $\frac{n}{\sqrt{2}}$ $\left(\frac{n}{\sqrt{2}}\right)^2$ $\left(\frac{n}{\sqrt{2}}\right)^3$ $\left(\frac{n}{\sqrt{2}}\right)^4$

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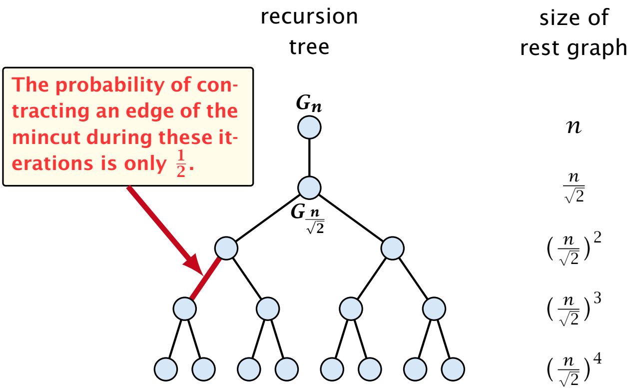

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Let for an edge e in the recursion tree, $h(e)$ denote the height (distance to leaf level) of the parent-node of e (end-point that is higher up in the tree). Let h denote the height of the root node.

Call an edge e alive if there exists a path from the parent-node of e to a descendant leaf, after we randomly deleted edges. Note that an edge can only be alive if it hasn't been deleted.

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The probability that an edge e is alive is at least $\frac{1}{h(e)+1}$.

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15 Global Mincut

Lemma 4
One run of the algorithm can be performed in time $\mathcal{O}(n^2 \log n)$ and has a success probability of $\Omega(\frac{1}{\log n})$.

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