4 Modelling Issues

What do you measure?

▶ Memory requirement
▶ Running time
▶ Number of comparisons
▶ Number of multiplications
▶ Number of hard-disc accesses
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Ernst Mayr, Harald Räcke 11. Apr. 2018
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How do you measure?

▶ Implementing and testing on representative inputs
  ▶ How do you choose your inputs?
  ▶ May be very time-consuming.
  ▶ Very reliable results if done correctly.
  ▶ Results only hold for a specific machine and for a specific set of inputs.

▶ Theoretical analysis in a specific model of computation.
  ▶ Gives asymptotic bounds like “this algorithm always runs in time $O(n^2)$”.
  ▶ Typically focuses on the worst case.
  ▶ Can give lower bounds like “any comparison-based sorting algorithm needs at least $\Omega(n \log n)$ comparisons in the worst case”.
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Input length
The theoretical bounds are usually given by a function \( f : \mathbb{N} \to \mathbb{N} \) that maps the input length to the running time (or storage space, comparisons, multiplications, program size etc.).

The input length may e.g. be
- the size of the input (number of bits)
- the number of arguments

Example 1
Suppose \( n \) numbers from the interval \( \{1, \ldots, N\} \) have to be sorted. In this case we usually say that the input length is \( n \) instead of e.g. \( n \log N \), which would be the number of bits required to encode the input.
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How to measure performance

1. Calculate running time and storage space etc. on a simplified, idealized model of computation, e.g. Random Access Machine (RAM), Turing Machine (TM).
2. Calculate number of certain basic operations: comparisons, multiplications, harddisc accesses.

Version 2 is often easier, but focusing on one type of operation makes it more difficult to obtain meaningful results.
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Turing Machine

- Very simple model of computation.
  - Only the “current” memory location can be altered.
  - Very good model for discussing computability, or polynomial vs. exponential time.
  - Some simple problems like recognizing whether input is of the form $x^2$, where $x$ is a string, have quadratic lower bound.

⇒ Not a good model for developing efficient algorithms.
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Random Access Machine (RAM)

- Input tape and output tape (sequences of zeros and ones; unbounded length).
- Memory unit: infinite but countable number of registers $R[0], R[1], R[2], \ldots$.
- Registers hold integers.
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Random Access Machine (RAM)

Operations

- input operations (input tape → $R[i]$)
  - READ $i$
- output operations ($R[i]$ → output tape)
  - WRITE $i$
- register-register transfers
  - $R[j] := R[i]$
  - $R[i] := R[j]$
- indirect addressing
  - $R[j] := R[R[i]]$
  loads the content of the $R[i]$-th register into the $j$-th register
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Random Access Machine (RAM)

Operations

- **branching (including loops) based on comparisons**
  - **jump x**
    - jumps to position x in the program;
    - sets instruction counter to x;
    - reads the next operation to perform from register $R[x]$.
  - **jumpz x R[i]**
    - jump to x if $R[i] = 0$
    - if not the instruction counter is increased by 1;
  - **jumpi i**
    - jump to $R[i]$ (indirect jump);

- **arithmetic instructions: +, −, ×, /**
  - $R[i] := R[j] + R[k]$
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Model of Computation

- **uniform cost model**
  Every operation takes time 1.

- **logarithmic cost model**
  The cost depends on the content of memory cells:
  - The time for a step is equal to the largest operand involved;
  - The storage space of a register is equal to the length (in bits) of the largest value ever stored in it.

Bounded word RAM model: cost is uniform but the largest value stored in a register may not exceed $2^w$, where usually $w = \log_2 n$. 

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Example 2

**Algorithm 1** RepeatedSquaring\((n)\)

1. \(r \leftarrow 2;\)
2. **for** \(i = 1 \rightarrow n\) **do**
3. \(r \leftarrow r^2\)
4. **return** \(r\)

- **running time:**
  - **uniform model:** \(n\) steps
  - **logarithmic model:** \(1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1 = \Theta(2^n)\)
- **space requirement:**
  - **uniform model:** \(O(1)\)
  - **logarithmic model:** \(O(2^n)\)
4 Modelling Issues

Example 2

Algorithm 1 RepeatedSquaring(n)

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- running time:
  - uniform model: $n$ steps
  - logarithmic model: $1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1 = \Theta(2^n)$

- space requirement:
  - uniform model: $\Theta(1)$
  - logarithmic model: $\Theta(2^n)$
4 Modelling Issues

Example 2

**Algorithm 1** RepeatedSquaring\((n)\)

1. \(r \leftarrow 2;\)
2. for \(i = 1 \rightarrow n\) do
3. \(r \leftarrow r^2\)
4. return \(r\)

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- **best-case complexity:**
  \[ C_{bc}(n) := \min\{C(x) \mid |x| = n\} \]

  Usually easy to analyze, but not very meaningful.

- **worst-case complexity:**
  \[ C_{wc}(n) := \max\{C(x) \mid |x| = n\} \]

  Usually moderately easy to analyze; sometimes too pessimistic.

- **average case complexity:**
  \[ C_{avg}(n) := \frac{1}{|I_n|} \sum_{|x|=n} C(x) \]

  more general: probability measure \( \mu \)

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  The algorithm may use random bits. Expected running time (over all possible choices of random bits) for a fixed input $x$. Then take the worst-case over all $x$ with $|x| = n$. 
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