Suppose you want to develop a data structure with:

- **Insert**(*x*): insert element *x*.
- **Search**(*k*): search for element with key *k*.
- **Delete**(*x*): delete element referenced by pointer *x*.
- **find-by-rank**(*ℓ*): return the *ℓ*-th element; return “error” if the data-structure contains less than *ℓ* elements.

Augment an existing data-structure instead of developing a new one.
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Augment an existing data-structure instead of developing a new one.
How to augment a data-structure

1. choose an underlying data-structure

2. determine additional information to be stored in the underlying structure

3. verify/show how the additional information can be maintained for the basic modifying operations on the underlying structure.

4. develop the new operations
7.4 Augmenting Data Structures

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Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $O(\log n)$.

1. We choose a red-black tree as the underlying data-structure.
2. We store in each node $v$ the size of the sub-tree rooted at $v$.
3. We need to be able to update the size-field in each node without asymptotically affecting the running time of insert, delete, and search. We come back to this step later...
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4. How does find-by-rank work?

Find-by-rank($k$) := Select(root,$k$) with

Algorithm 11 Select($x$, $i$)

1: if $x$ = null then return error
2: if left[$x$] ≠ null then $r$ ← left[$x$].size + 1 else $r$ ← 1
3: if $i = r$ then return $x$
4: if $i < r$ then
5: return Select(left[$x$], $i$)
6: else
7: return Select(right[$x$], $i - r$)
Select($x, i$)

Find-by-rank:

- decide whether you have to proceed into the left or right sub-tree
- adjust the rank that you are searching for if you go right
Select($x, i$)

Select(25, 14)

Find-by-rank:

- decide whether you have to proceed into the left or right sub-tree
- adjust the rank that you are searching for if you go right
Select\((x, i)\)

Select\((13, 14)\)

Find-by-rank:

- decide whether you have to proceed into the left or right sub-tree
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Select \((x, i)\)

Select \((21, 5)\)

Find-by-rank:

- decide whether you have to proceed into the left or right sub-tree
- adjust the rank that you are searching for if you go right
Select \((x, i)\)

Select \((16, 5)\)

**Find-by-rank:**

- decide whether you have to proceed into the left or right sub-tree
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Select($x, i$)

Find-by-rank:

- decide whether you have to proceed into the left or right sub-tree
- adjust the rank that you are searching for if you go right
Select($x, i$)

Select($20, 1$)

Find-by-rank:
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3. How do we maintain information?

Search($k$): Nothing to do.

Insert($x$): When going down the search path increase the size field for each visited node. Maintain the size field during rotations.

Delete($x$): Directly after splicing out a node traverse the path from the spliced out node upwards, and decrease the size counter on every node on this path. Maintain the size field during rotations.
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Rotations

The only operation during the fix-up procedure that alters the tree and requires an update of the size-field:

\[
\text{LeftRotate}(x) \quad \text{RightRotate}(z)
\]

The nodes \( x \) and \( z \) are the only nodes changing their size-fields. The new size-fields can be computed \textit{locally} from the size-fields of the children.