Technische Universität München
Fakultät für Informatik
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Winter Semester 2017/18
Problem Set 4
November 13, 2017

Online and Approximation Algorithms

Due November 20, 2017 at 10:00

Exercise 1 (RMARK – 10 points)

Show an example with corresponding request sequence such that the cost of RMARK is always greater than H_k times the cost of the optimal offline algorithm, independent of the random choices.
Hint: An example exists consisting of only 4 pages and a fast memory of size 2.

Exercise 2 (LRU Analysis with Potential Function – 10 points)

Show that LRU is k-competitive by using the potential method, where k is the number of pages that fit in the fast memory.

Hint: Let $S_{\text{LRU}}(t)$ be the set of pages in LRU's fast memory after the t-th request and let $S_{\text{OPT}}(t)$ be the set of pages in OPT's fast memory. Assign the values $\{1, \ldots, k\}$ to the pages of $S_{\text{LRU}}(t)$ in the order of the last requests, such that the least recently requested page has the value 1 and the most recently requested page has value k. The assigned value of page p is denoted by $w_t(p)$. Let $S(t) \coloneqq S_{\text{LRU}}(t) \setminus S_{\text{OPT}}(t)$. Use the following potential function for your analysis:

$$\Phi(t) = \sum_{p \in S(t)} w_t(p)$$

Exercise 3 (Randomized Ski Rental, Lower Bound – 10 points)

Use Yao's Minimax Principle to prove that the competitive ratio of any randomized algorithm for the ski rental problem is lower bounded by $(1+x)$, where $x \in [\frac{1}{7}, \frac{1}{3}]$ is a constant of your choice.

Exercise 4 (Randomized Sorting – 10 points)

Consider an array with n integers that should be sorted with a randomized comparison-based algorithm. Use Yao's Minimax Principle to prove that the expected number of comparisons is $\Omega(n \log n)$.