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## Online and Approximation Algorithms

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*Due December 18, 2017 at 10:00*

### **Exercise 1 (Navigation on a line – 10 points)**

Propose a randomized algorithm for the navigation problem on a line and show that it is 7-competitive.

### **Exercise 2 (Room problem – 10 points)**

In the lecture we saw an algorithm for the square room problem that achieves a competitive ratio of  $\sqrt{n}$ .

Modify the algorithm as well as its analysis for rooms of dimension  $2N \times 2n$  where  $N \neq n$ . The starting point is  $s = (0, 0)$ , the target  $t = (N, n)$ .

### **Exercise 3 (Exploration on cellular environments – 10 points)**

Consider a robot which moves in a rectangular grid environment similar to a chessboard. Each cell of the environment is either *free* and can be visited by the robot, or *blocked* and unaccessible by the robot. In one step, the robot is in some cell and it moves in one of the 8 neighboring cells. Of course, the new cell must be free. Starting from a cell  $s$ , we want to visit all the empty cells of the environment and return back to  $s$  assuming that the environment is not known by the robot in advance. The objective is to perform a minimum number of steps. Checking if one of the neighboring cells is free or blocked does not cost anything. Propose an online algorithm and show that it is 2-competitive.

### **Exercise 4 (Spiral – 10 points)**

Imagine yourself standing on a 2-dimensional grid, searching for a point  $t = (t_v, t_h)$  where  $t_v$  and  $t_h$  are the vertical and horizontal distances from your current position. You do know that  $t$  is finite, however you do not know the exact values for  $t_v$  and  $t_h$ . Develop an algorithm for reaching  $t$  and prove its competitiveness.