Technische Universität München
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Winter Semester 2017/18
Problem Set 11
January 22, 2018

Online and Approximation Algorithms

Due January 29, 2018 at 10:00

Exercise 1 (Fractional Knapsack – 10 points)

In the Fractional Knapsack problem, we are given n items with weights $w_1, w_2, \dots, w_n$ and values $v_1, v_2, \dots, v_n$. The objective is to pack items of maximum total value in a knapsack of capacity B and we are allowed to take any fraction of each item. A fraction $x_i \in [0,1]$ of item i has weight $x_i w_i$ and value $x_i v_i$. Propose an optimal algorithm for the Fractional Knapsack problem and show that it solves the problem optimally in polynomial time.

Exercise 2 (Any Fit Strategies – 10 points)

In the Bin Packing problem, we are given n elements of size $a_1, \dots, a_n \in [0,1]$. The objective is to pack the elements into bins without exceeding their capacity, such that the number of used bins is minimized.
Prove that *Any Fit* (the set of strategies that only open a new bin if the item does not fit in any currently open bin) achieves an approximation factor of 2.

Exercise 3 (Lower bound for Bin Packing – 10 points)

Consider the Partition problem. Given n non-negative numbers $a_1, a_2, \dots, a_n$ such that $\sum_{i=1}^{n} a_i = A$. The Partition problem is to decide whether it is possible to identify a subset $B \subset A$ such that $\sum_{i \in B} a_i = \frac{1}{2}A$. This problem is NP-hard.
Use a reduction from the Partition problem to show that there is no polynomial-time approximation algorithm that can approximate Bin Packing to a ratio better than 3/2.

Exercise 4 (Makespan Minimization on 2 Machines – 10 points + 5 bonus points)

We consider the problem of scheduling n jobs with processing times $p_1, p_2, \dots, p_n$ on two machines with the objective of minimizing the makespan. Our aim is to derive a PTAS (different than the one presented in class).

1. Let M be a boolean 2-dimensional matrix. For $i = 0, 1, \dots, n$ and $j = 0, 1, \dots, P$, where $P = \sum_{k=1}^{n} p_k$, $M(i,j)$ is true if there is a subset $S \subseteq \{1, 2, \dots, i\}$ such that $\sum_{k \in S} p_k = j$. Otherwise, $M(i,j)$ is false. Propose an algorithm for computing M in pseudo-polynomial time.

2. Then, for a given pair (i,j) and if $M(i,j)$ is true, propose an algorithm for computing a subset $S \subseteq \{1, 2, \dots, i\}$ such that $\sum_{k \in S} p_k = j$ in pseudo-polynomial time.

3. Propose an algorithm for solving the makespan minimization problem on two machines in pseudo-polynomial time.

4. **Bonus:** By using an appropriate scaling the processing times of the jobs, show that there exists a PTAS which computes an $(1+\epsilon)$-approximation algorithm in $O(\frac{n^2}{\epsilon})$ time.