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Winter Semester 2018/19
Problem Set 1
October 23, 2018

Advanced Algorithms

Due October 30, 2018 at 10:00

Note: You are welcome to submit in groups of two. If you wish to submit individually, then Exercises 1 and 3 are to be solved.

Exercise 1 (Max/Min – 10 points)

Let $\mathcal{N} := \{x_1, x_2, \dots, x_n\}, n = 2^k, k \in \mathbb{N}, k > 0$, be a set of natural numbers. Our objective is to find the maximum number and the minimum number in $\mathcal{N}$.

- How many comparisons are needed in an iterative approach? Describe the corresponding procedure and argue how many comparisons are needed.
- Now use the *divide-and-conquer* paradigm for developing an algorithm that finds both the maximum and minimum using at most $(\frac{3}{2}n - 2)$ comparisons. Describe the pseudocode of the algorithm and verify the required number of comparisons.

Exercise 2 (Closest pair of points – 10 points)

Let $\mathcal{Q}$ be a set of $n \geq 2$ points in the plane. The L_p-*distance* between points $v_1 := (x_1, y_1)$ and $v_2 := (x_2, y_2)$ is given by the expression $\|v_1 - v_2\|_p := (|x_1 - x_2|^p + |y_1 - y_2|^p)^{1/p}$ for $0 < p < \infty$ and $\|v_1 - v_2\|_\infty := \max(|x_1 - x_2|, |y_1 - y_2|)$. We have studied the closest pair algorithm in the class for Euclidean distance which is L_2 distance. Modify the closest-pair algorithm when we consider the distance between points to be L_1-distance (also called Manhattan distance) and the L_∞-distance (also called Chebyshev distance), respectively.

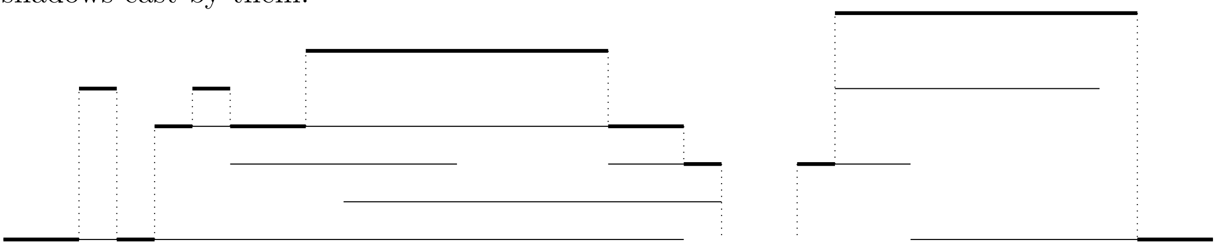
Exercise 3 (Balanced partition – 10 points)

Let G be any binary tree with n vertices with n to be an even number.

- Show that by removing a single edge, we can partition the vertices of G into two sets $\mathcal{X}$ and $\mathcal{Y}$ such that there are no edges between the sets $\mathcal{X}$ and $\mathcal{Y}$ and $n/4 \leq |\mathcal{X}| \leq |\mathcal{Y}| \leq 3n/4$.
- Give an example of a simple binary tree whose most evenly balanced partition upon removal of a single edge has $|\mathcal{Y}| = 3n/4$.
- Show that for any $k = 1, 2, \dots, n-1$, by removing at most $3\log_2(n)$ edges, we can partition the vertices of G into two sets $\mathcal{X}, \mathcal{Y}$ such that there are no edges between the sets $\mathcal{X}$ and $\mathcal{Y}$ and $|\mathcal{X}| = k$ and $|\mathcal{Y}| = n - k$.

Exercise 4 (Upper outline – 10 points)

In the game Superior Maria the protagonist Maria traverses levels consisting of various platforms on her pursue to divide and conquer Toadstool Country. Formally platforms correspond to closed line segments parallel to the x-axis. Some parts of platforms are illuminated by the sun and are fairly safe to traverse while in the more shadowy parts various dangers lurk. Meteorological studies of Toadstool Country yield that a part of a platform is illuminated if and only if there is no other platform directly above it. Below a sample level is given with the illuminated parts highlighted. The dotted lines outline the shadows cast by them.



Your job is to help Maria by giving an algorithm which computes a list of all those illuminated parts in time $O(n \log_2(n))$ where n is the total number of platforms. Describe the pseudocode of the algorithm and argue why the running time is $O(n \log_2(n))$.