Technische Universität München
Fakultät für Informatik
Lehrstuhl für Theoretische Informatik
Prof. Dr. Susanne Albers
Dr. Arindam Khan
Maximilian Janke

Winter Semester 2018/19
Problem Set 2
October 30, 2017

Advanced Algorithms

Due November 6, 2018 at 10:00

Note: You are welcome to submit in groups of two. If you wish to submit individually, then Exercises 1 and 2 are to be solved.

Exercise 1 (FFT Computation – 10 points)

Compute the product of the two polynomials using the Fast Fourier Transform (FFT):

- $p(x) = 5x + 2$,
- $q(x) = 6x + 1$.

Specify all (recursive) calls of the FFT algorithm as well as the outputs and the assignments of the temporary variables used during the execution.

Exercise 2 (FFT for SetSum – 10 points)

Let S and T be two sets of integers in the range $[0, z-1]$ where z is a power of two. Use a single DFT to compute the following in $O(z \log_2 z)$ time:

1. all elements contained in the set $S + T := \{s + t : s \in S, t \in T\}$,
2. for each element $u \in S + T$, the number $k_u := |\{(s,t) \in S \times T : s + t = u\}|$.

Hint: Find some polynomials p_S, p_T of degree $< z$ that represents the sets S and T.

Exercise 3 (D&C Multiplication – 10 points)

1. Let $px + r$ and $qx + s$ be two linear polynomials. Show how to multiply them using only three multiplications.
   Hint: One of the multiplications is $\big((p + r) \cdot (q + s)\big)$.
2. Give a divide-and-conquer algorithm for multiplying two polynomials of degree at most n in time $O(n^{\log_2 3})$.

Exercise 4 (D&C Median – 10 points)

We are given two sets A and B containing integers and both sets are stored as (increasingly) ordered arrays. Our goal is to compute the median of their disjoint union $A \amalg B$. Show how to do that in time $O(\log(|A| + |B|))$ using a divide-and-conquer based algorithm.