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Problem Set 3
November 6, 2018

Advanced Algorithms

Due November 15, 2018 at 12:00

Note: You are welcome to submit in groups of two. If you wish to submit individually either exercise may be solved.

Exercise 1 (Randomized Quicksort – 10 points)

Consider the following way to pick a random permutation π of the set of $n(> 2)$ integers $\mathcal{I}_n := \{1, 2, \dots, n\}$:

- Run Randomized Quicksort (RandQuick).
- Let T be the recursion tree corresponding to the execution of Randomized Quicksort.
- Let π be the permutation induced by the level-order traversal of T (i.e., the nodes are visited in the increasing order of level numbers and in a left-to-right order within each level).

Is π uniformly distributed over the space of all permutations of the elements in $\mathcal{I}_n$? Explain your answer.

Exercise 2 (Another Euclidean algorithm – 10 points)

Let us consider the set $R = \{a + bi \mid a, b \in \mathbb{Z}\} \subseteq \mathbb{C}$. Recall that the square of the absolute value of an element $x = a + bi \in R$ is given by $|x|^2 = a^2 + b^2$. Given two elements $x, y \in R$ an element g is called their greatest common divisor if both elements are divisible by g, i.e. for some $\tilde{x}, \tilde{y} \in R$ we have $x = \tilde{x}g$ and $y = \tilde{y}g$, and if there are integers $s, t \in R$ such that $g = sx + ty$.
Show, that given $x, y \in R \setminus \{0\}$, we can always compute elements $p, q \in R$ such that $x = py + q$ and $|q|^2 \leq \frac{|y|^2}{2}$ holds. In the lecture we studied an algorithm attributed to Euclid to compute the greatest common divisor of two integers. Now adapt the algorithm to compute a greatest common divisor g of x and y using at most $O(\log(|y|^2))$ arithmetic operations.