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Winter Semester 2018/19
Problem Set 5
November 27, 2018

Advanced Algorithms

Due December 4, 2018 at 10:00

Note: You are welcome to submit in groups of two. If you wish to submit individually, you may solve either Problem 1 or 2.

Exercise 1 (Rotations in Treaps – 10 points)

The *left spine* of a binary search tree is the path from the root to the node with the smallest key. In other words, the left spine is the path from the root that consists of only left edges. Symmetrically, the *right spine* is the path from the root consisting only right edges. The *length* of a spine is the number of nodes it contains (including the root).

1. Consider a treap T immediately after inserting node x. Let C be the length of the right spine of the left subtree of x. Let D be the length of the left spine of the right subtree of x. Argue that the total number of rotations that were performed during the insertion of x is equal to $C + D$.

2. Show that for nodes x and $y(\neq x)$, y is in the right spine of the left subtree of x if and only if $\mathit{prio}(x) < \mathit{prio}(y)$, $\mathit{key}(x) > \mathit{key}(y)$, and for every z such that $\mathit{key}(y) < \mathit{key}(z) < \mathit{key}(x)$, there is $\mathit{prio}(z) > \mathit{prio}(y)$.

Exercise 2 (Distinct Min-cuts – 10 points)

Consider the randomized min cut algorithm presented in the class. We showed that, for any graph G with n vertices, the probability that the algorithm finds a specific min cut C of G is at least $\frac{2}{n(n-1)}$.

1. What can we say about the maximum number of distinct min cuts that a graph G can have?

2. Give an example of a graph (with n vertices) with maximum number of distinct min cuts.

3. Use the randomized edge contraction algorithm to find *all* the global minimum cuts in any graph G.