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Fakultät für Informatik
Lehrstuhl für Theoretische Informatik
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Winter Semester 2018/19
Problem Set 6
December 4, 2018

Advanced Algorithms

Due December 11, 2018 at 10:00

Note: You are welcome to submit in groups of two. If you wish to submit individually, you may solve any two of the exercises.

Exercise 1 (Min-cuts with vertex merging – 10 points)

Consider the randomized min cut algorithm presented in the class. Suppose that at each step of the min-cut algorithm, instead of choosing a random edge for contraction, two vertices are chosen at random and are merged into a single vertex. Show that there exist inputs for which the probability that the modified algorithm finds a min cut is exponentially small.

Exercise 2 (Min k-cut – 10 points)

Given an unweighted and undirected graph $G := (V, E)$, a 3-*cut* is a partition of V into three nonempty sets A, B, C. The size of the cut is the number of edges connecting vertices from different sets. Extend the randomized min cut algorithm presented in the class to give an algorithm to find a minimum 3-cut. Extend your answer to provide an algorithm for the case of minimum k-cuts for any constant integer $k \geq 3$.

Exercise 3 (Min s-t-cut – 10 points)

Let $G := (V, E)$ be a weighted undirected graph with $|V| \geq 3$. Show that there are at least two distinct vertices $t_1, t_2 \in V$ such that $(V \setminus \{t_1\}, \{t_1\})$ is a minimum s_1-t_1-cut, for some $s_1 \in V$, and $(V \setminus \{t_2\}, \{t_2\})$ is a minimum s_2-t_2-cut, for some $s_2 \in V$.
Find graphs in which there are no three such vertices.

Exercise 4 (Minimum cardinaliy min-cut – 10 points)

Let $G = (V, E, c)$ be a network with integer capacities c. The problem is to find a cut in G of minimum capacity that has the smallest number of edges among all minimum capacity cuts of G. Show how to modify the capacities of G to create a new network $G' = (V, E, c')$ such that any minimum capacity cut in G' is a minimum capacity cut in G with the smallest number of edges. Prove the correctness of your construction.