Technische Universität München
Fakultät für Informatik
Lehrstuhl für Theoretische Informatik
Prof. Dr. Susanne Albers
Dr. Arindam Khan
Maximilian Janke
Luisa Peter

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Problem Set 9
January 8, 2019

Advanced Algorithms

Due January 15, 2019 at 10:00

Note: You are welcome to submit in groups of two. If you wish to submit individually, you should solve exercise 1.

Exercise 1 (Fibonacci Heaps – 10 points)

Execute the following operations on an initially empty Fibonacci heap:

$$\begin{aligned}&\mathit{insert}(15),\ \mathit{insert}(27),\ \mathit{insert}(6),\ \mathit{insert}(34),\ \mathit{insert}(42),\\&\mathit{insert}(35),\ \mathit{insert}(3),\ \mathit{insert}(41),\ \mathit{insert}(22),\ \mathit{insert}(12),\\&\mathit{deletemin}(),\ \mathit{decreasekey}(27,2),\ \mathit{decreasekey}(34,17),\ \mathit{deletemin}().\end{aligned}$$

For all intermediate stages, illustrate the structure of the Fibonacci heap, fill in the key values and possible marks of the nodes and tag the current minimum. A new element shall always be inserted to the right of the current minimum. The consolidation during the operation *deletemin* shall start with the element to the right of the deleted minimum.

Exercise 2 (Generalized Cascading-Cut – 10 points)

Suppose we generalize the cascading-cut rule to cut a node v from its parent as soon as it loses its kth child, for some integer constant k. (The rule from the lecture uses $k = 2$.) For what values of k is the maximum rank of any node in a Fibonacci heap with n nodes in $O(\log_2 n)$?

Hint: You may use the fact that a recursion for a sequence (a_n) of the form

$$a_n = c_{k-1}a_{n-1} + c_{k-2}a_{n-2} + \cdots + c_0 a_{n-k}$$

(i.e. a linear, homogeneous recursion with constant coefficients) has an explicit general solution in terms of the roots of its characteristic polynomial

$$x^k - (c_{k-1}x^{k-1} + c_{k-2}x^{k-2} + \cdots + c_0).$$