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Assignment 1
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# Efficient Algorithms and Data Structures I

*Deadline: None, Tutorial exercises only.*

## Tutorial Exercise 1

For constants $c > 0$, $0 < \varepsilon < 1/2$ and $k > 1$, arrange the following functions of $n$ in non-decreasing asymptotic order so that $f_i(n) \in O(f_{i+1}(n))$ for two consecutive functions in your sequence. Also indicate whether $f_i(n) \in \Theta(f_{i+1}(n))$ holds or not.

$$n^k, n^{1+sin(n)}, \log(n!), n^{k+\varepsilon}, n^n, n, n^k(\log n)^c, n!, 2^n, 3^n, n\log\log n, n\log(n), n^\varepsilon, n^{1/\log n}$$

Use any base for the logarithm you like the most. The natural logarithm facilitates the calculations.

## Tutorial Exercise 2

Show by using the basic definition of the $\Theta$-notation, that for any real constants $a$ and $b$, where $b > 0$,

$$(n+a)^b = \Theta(n^b)$$

## Tutorial Exercise 3

Jonathan is frustrated. His boss has given him a list of integers $a_1, \ldots, a_n$ with a weird assignment: For each $i \in [n]$, compute the product $b_i = \prod_{j\in[n], j\neq i} a_j$. Unfortunately, the latest update of his operating system has broken the division operator on his machine.

(a) At first, Jonathan's logarithm operator is still working well (in $\mathcal{O}(1)$ time). Find a way for him to compute all $b_i$'s in $\mathcal{O}(n)$ steps.

(b) The next update to his system breaks the logarithm operator as well. Help Jonathan find a new $\mathcal{O}(n)$ time algorithm to compute all $b_i$'s.

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The advanced reader who skips parts
that appear too elementary may miss
more than the less advanced reader
who skips parts that appear too com-
plex.
                          - G. Pòlya
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