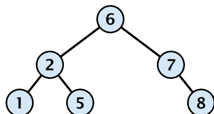
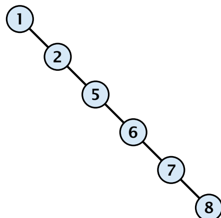
7.1 Binary Search Trees

An (internal) binary search tree stores the elements in a binary tree. Each tree-node corresponds to an element. All elements in the left sub-tree of a node v have a smaller key-value than $\text{key}[v]$ and elements in the right sub-tree have a larger-key value. We assume that all key-values are different.

(External Search Trees store objects only at leaf-vertices)

Examples:



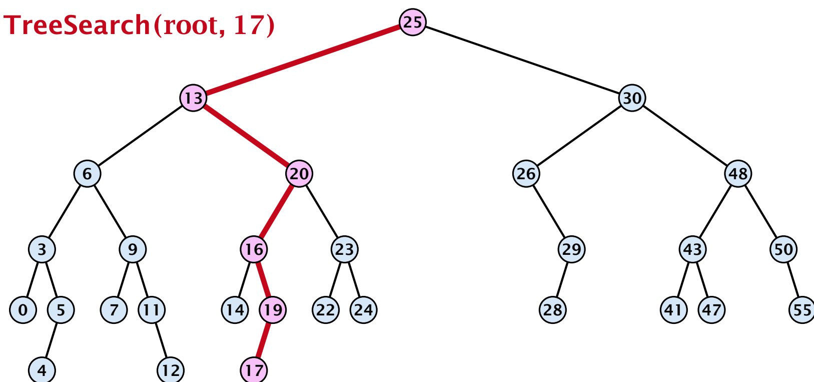


7.1 Binary Search Trees

We consider the following operations on binary search trees. Note that this is a super-set of the dictionary-operations.

- $T.\text{insert}(x)$
- $T.\text{delete}(x)$
- $T.\text{search}(k)$
- $T.\text{successor}(x)$
- $T.\text{predecessor}(x)$
- $T.\text{minimum}()$
- $T.\text{maximum}()$

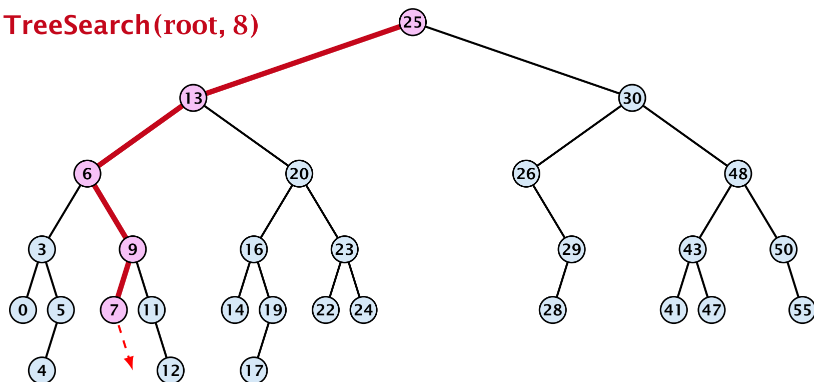
Binary Search Trees: Searching



Algorithm 1 TreeSearch(x, k)

1: **if** $x = \text{null}$ **or** $k = \text{key}[x]$ **return** x
2: **if** $k < \text{key}[x]$ **return** TreeSearch$(\text{left}[x], k)$
3: **else return** TreeSearch$(\text{right}[x], k)$

Binary Search Trees: Searching



Algorithm 1 TreeSearch(x, k)

1: **if** x = null **or** $k = \text{key}[x]$ **return** x
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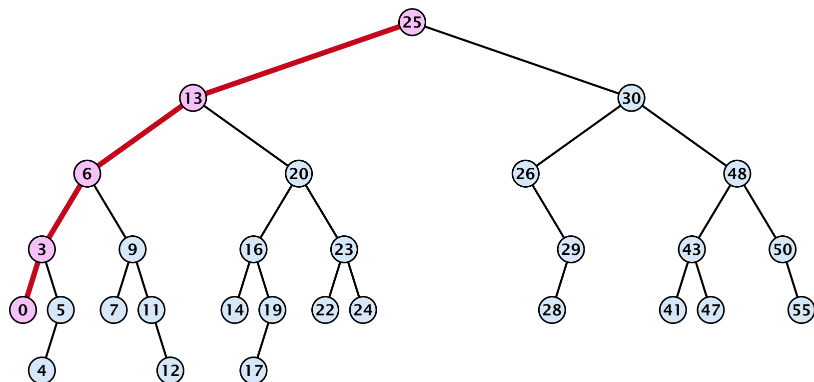
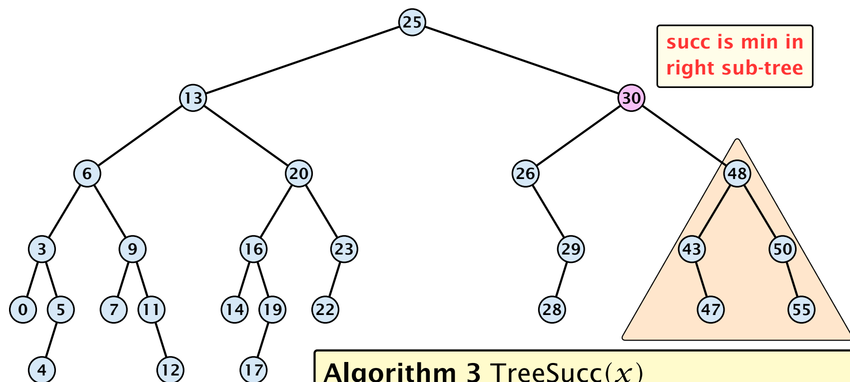
Binary Search Trees: Minimum



Algorithm 2 TreeMin(x)

1: **if** $x = \text{null}$ **or** $\text{left}[x] = \text{null}$ **return** x
2: **return** TreeMin($\text{left}[x]$)

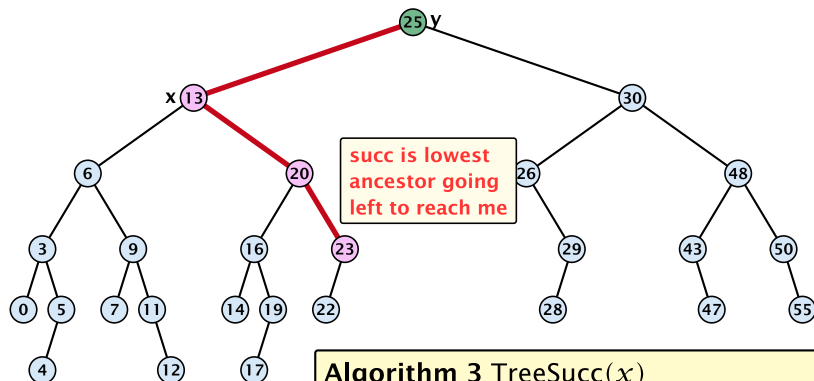
Binary Search Trees: Successor



Algorithm 3 TreeSucc(x)

1: **if** right$[x] \neq$ null **return** TreeMin(right$[x]$)
2: $y \leftarrow$ parent$[x]$
3: **while** $y \neq$ null **and** $x =$ right$[y]$ **do**
4: $\quad x \leftarrow y; y \leftarrow$ parent$[x]$
5: **return** y;

Binary Search Trees: Successor



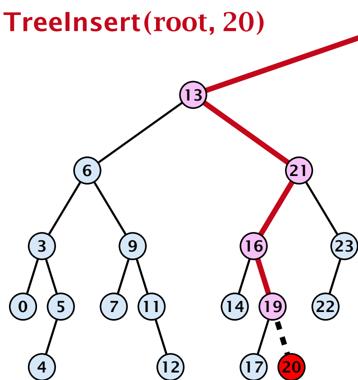
Algorithm 3 TreeSucc(x)

1: **if** right[x] $\neq$ null **return** TreeMin(right[x])
2: $y \leftarrow$ parent[x]
3: **while** $y \neq$ null **and** $x =$ right[y] **do**
4: $\quad x \leftarrow y; y \leftarrow$ parent[x]
5: **return** y;

Binary Search Trees: Insert

Insert element **not** in the tree.

TreeInsert(root, 20)

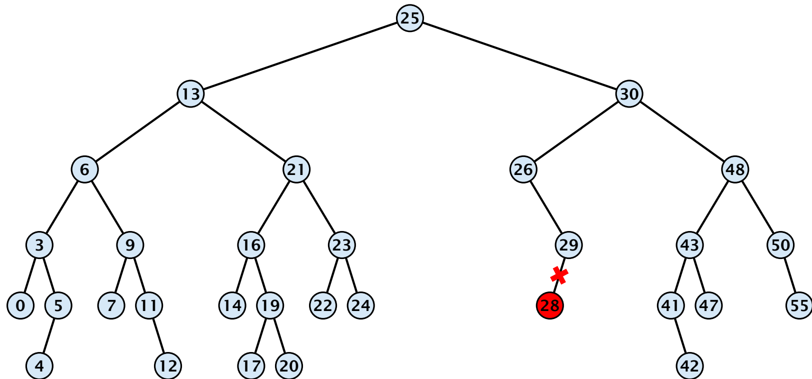


Search for z. At some point the search stops at a null-pointer. This is the place to insert z.

Algorithm 4 TreeInsert(x, z)

1: **if** x = null **then**
2:     root$[T] \leftarrow z$; parent$[z] \leftarrow$ null;
3:     **return**;
4: **if** key$[x] >$ key$[z]$ **then**
5:     **if** left$[x]$ = null **then**
6:         left$[x] \leftarrow z$; parent$[z] \leftarrow x$;
7:     **else** TreeInsert(left$[x], z$);
8: **else**
9:     **if** right$[x]$ = null **then**
10:         right$[x] \leftarrow z$; parent$[z] \leftarrow x$;
11:     **else** TreeInsert(right$[x], z$);

Binary Search Trees: Delete

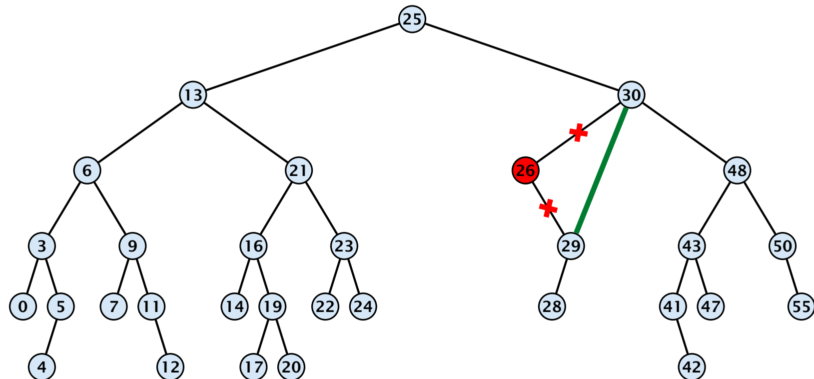


Case 1:

Element does not have any children

- Simply go to the parent and set the corresponding pointer to null.

Binary Search Trees: Delete

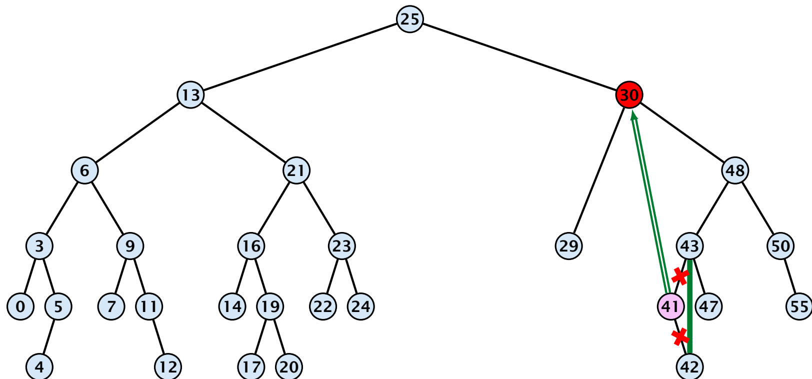


Case 2:

Element has exactly one child

- Splice the element out of the tree by connecting its parent to its successor.

Binary Search Trees: Delete



Case 3:

Element has two children

- Find the successor of the element
- Splice successor out of the tree
- Replace content of element by content of successor

Binary Search Trees: Delete

Algorithm 9 TreeDelete(z)

```
1:  if left[z] = null or right[z] = null
2:      then y ← z else y ← TreeSucc(z);          select y to splice out
3:  if left[y] ≠ null
4:      then x ← left[y] else x ← right[y];  x is child of y (or null)
5:  if x ≠ null then parent[x] ← parent[y];      parent[x] is correct
6:  if parent[y] = null then                      ⎫
7:      root[T] ← x                               ⎪
8:  else                                          ⎪
9:      if y = left[parent[y]] then               ⎬ fix pointer to x
10:         left[parent[y]] ← x                   ⎪
11:     else                                      ⎪
12:         right[parent[y]] ← x                  ⎭
13: if y ≠ z then copy y-data to z
```

Balanced Binary Search Trees

All operations on a binary search tree can be performed in time $\mathcal{O}(h)$, where h denotes the height of the tree.

However the height of the tree may become as large as $\Theta(n)$.

Balanced Binary Search Trees
With each insert- and delete-operation perform local adjustments to guarantee a height of $\mathcal{O}(\log n)$.

AVL-trees, Red-black trees, Scapegoat trees, 2-3 trees, B-trees, AA trees, Treaps

similar: SPLAY trees.

Binary Search Trees (BSTs)

Bibliography

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[CLRS90] Thomas H. Cormen, Charles E. Leiserson, Ron L. Rivest, Clifford Stein:
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MIT Press and McGraw-Hill, 2009

Binary search trees can be found in every standard text book. For example Chapter 7.1 in [MS08] and Chapter 12 in [CLRS90].