Technische Universität München
Fakultät für Informatik
Lehrstuhl für Algorithmen und Komplexität
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Winter term 2019/20
Assignment 03
October 28, 2019

Efficient Algorithms and Data Structures I

Deadline: November 4, 10:15 am in the ***Efficient Algorithms*** *mailbox.*

Homework 1 (5 Points)

Give tight asymptotic upper and lower bounds for the following recurrence relations.

(a) $T(n) = 9T(n/3) + n\sqrt{n} + n\log n$.

(b) $T(n) = 2T(n/4) + \sqrt{n}\log_2 n$.

(c) $T(n) = 4T(n/2) + n!$

Homework 2 (5 Points)

Given two $n\times n$ matrices A and B where n is a power of 2, we know how to find $C = A\cdot B$ by performing n^3 multiplications. Now let us consider the following approach. We partition A, B and C into equally sized block matrices as follows:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \qquad C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Consider the following matrices:

$$\begin{aligned} M_1 &= (A_{11} + A_{22})\cdot(B_{11} + B_{22}) \\ M_2 &= (A_{21} + A_{22})\cdot B_{11} \\ M_3 &= A_{11}\cdot(B_{12} - B_{22}) \\ M_4 &= A_{22}\cdot(B_{21} - B_{11}) \\ M_5 &= (A_{11} + A_{12})\cdot B_{22} \\ M_6 &= (A_{21} - A_{11})\cdot(B_{11} + B_{12}) \\ M_7 &= (A_{12} - A_{22})\cdot(B_{21} + B_{22}) \end{aligned}$$

Then,

$$C_{22} = M_1 - M_2 + M_3 + M_6 \ .$$

(a) Construct the matrices C_{11}, C_{12}, C_{21} from the matrices M_i, as demonstrated for C_{22}.

(b) Design an efficient algorithm for multiplying two $n\times n$ matrices based on these facts. Analyze its running time.

Homework 3 (5 Points)

Show tight asymptotic upper and lower bounds for $T(n)$, where $T(0)$ is an arbitrary constant, for the following recurrence relations

(a) $T(n) = T(n/2) + T(n/4) + T(n/8) + n$. Show $T(n) \in \Theta(n)$.

(b) $T(n) = T(n-2) + 2\ln n$. Show $T(n) \in \Theta(n \ln n)$.

As argued in the lecture you may assume that function arguments are always integer.
Hint: You may use without proof that $\ln(n+1) < \frac{1}{n} + \ln n$.

Homework 4 (5 Points)

The recursion $T(n)$ is

$$T(n) = \sqrt{n}T(\sqrt{n}) + n \quad .$$

Assuming that $T(n)$ is constant for sufficiently small n, show by induction that $T(n) = \Theta(n \log_2 \log_2 n)$.

Tutorial Exercise 1

The H-graph of order 0 is just a simple node. The H-graphs of order 1, 2, 3, and 4 are shown in Figure 1, Figure 2, Figure 3, and Figure 4, respectively. Let $f(\ell)$ denote the number of vertices of an H-graph of order ℓ. Develop a recurrence relation for f and solve your relation using techniques from the lecture.

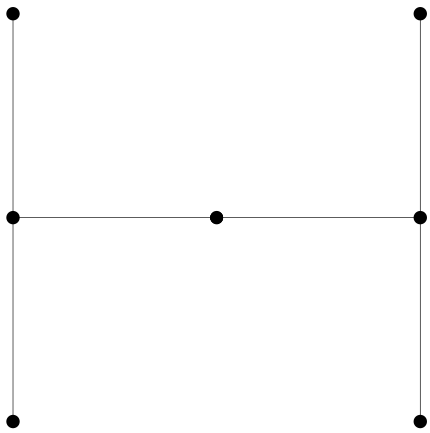

Figure 1: H-graph of order 1

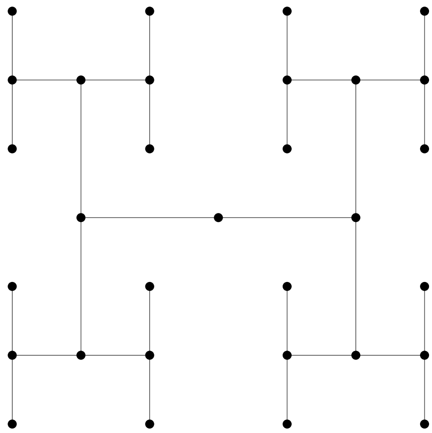

Figure 2: H-graph of order 2

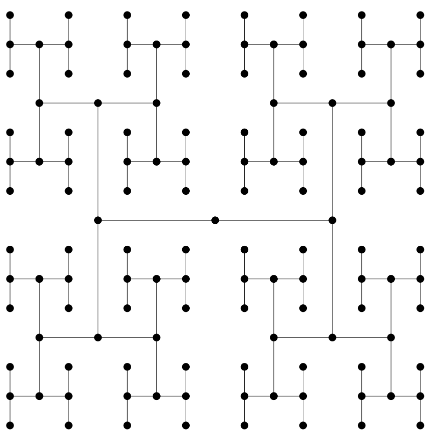

Figure 3: H-graph of order 3

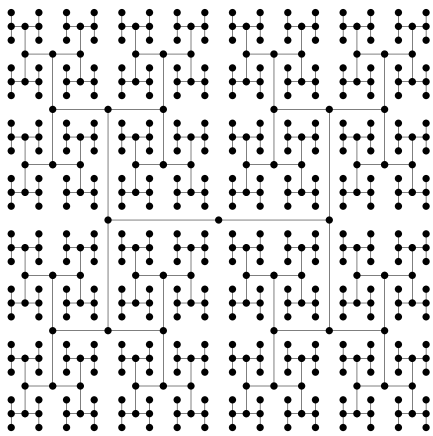

Figure 4: H-graph of order 4

I like trees because they seem more resigned to the way they have to live than other things do.
- W. Cather