

7.4 Augmenting Data Structures

Suppose you want to develop a data structure with:

- ▶ **Insert(x)**: insert element x .
- ▶ **Search(k)**: search for element with key k .
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- ▶ **find-by-rank(ℓ)**: return the ℓ -th element; return “error” if the data-structure contains less than ℓ elements.

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Augment an existing data-structure instead of developing a new one.

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3. verify/show how the additional information can be maintained for the basic modifying operations on the underlying structure.
4. develop the new operations

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Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $\mathcal{O}(\log n)$.

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3. We need to be able to update the size-field in each node without asymptotically affecting the running time of insert, delete, and search. We come back to this step later...

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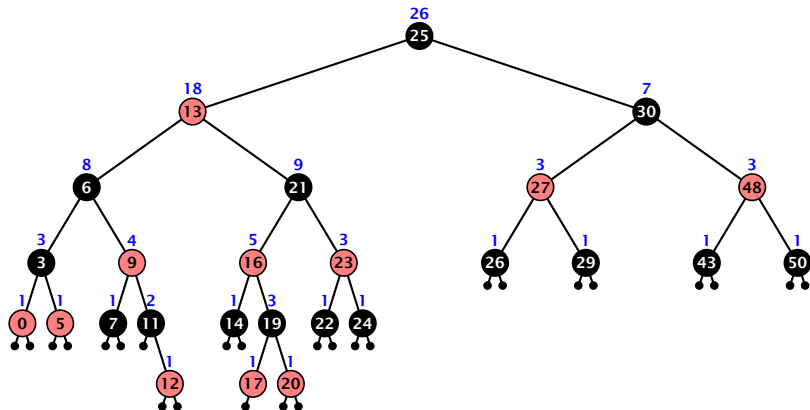
4. How does find-by-rank work?

Find-by-rank(k) := Select(root, k) with

Algorithm 1 Select(x, i)

```
1: if  $x = \text{null}$  then return error
2: if  $\text{left}[x] \neq \text{null}$  then  $r \leftarrow \text{left}[x].\text{size} + 1$  else  $r \leftarrow 1$ 
3: if  $i = r$  then return  $x$ 
4: if  $i < r$  then
5:     return Select( $\text{left}[x], i$ )
6: else
7:     return Select( $\text{right}[x], i - r$ )
```

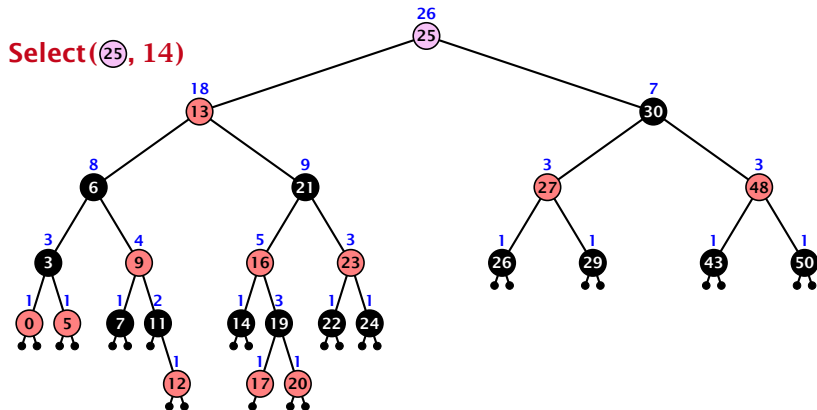
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Find-by-rank:

- ▶ decide whether you have to proceed into the left or right sub-tree
- ▶ adjust the rank that you are searching for if you go right

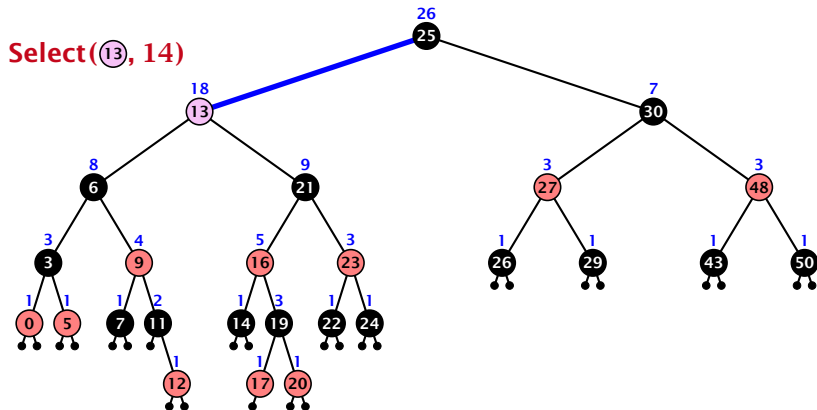
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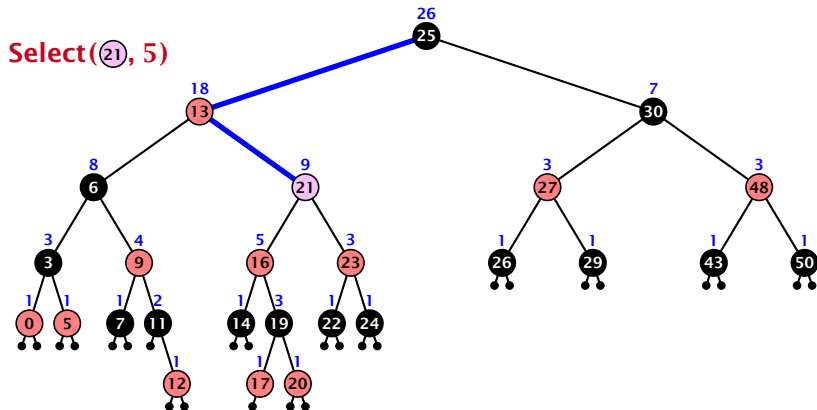
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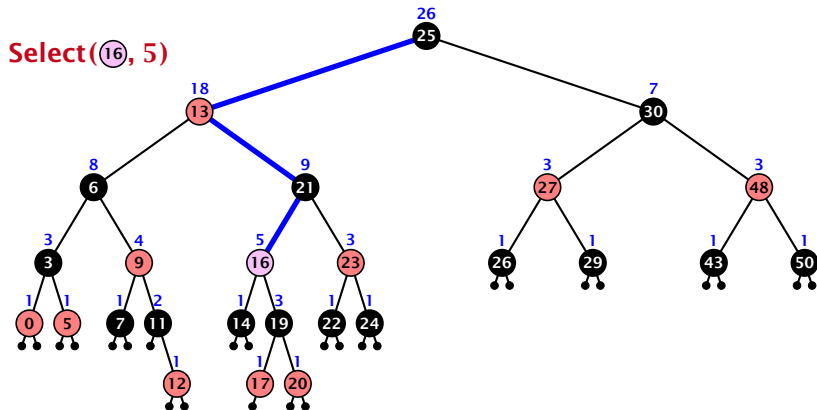
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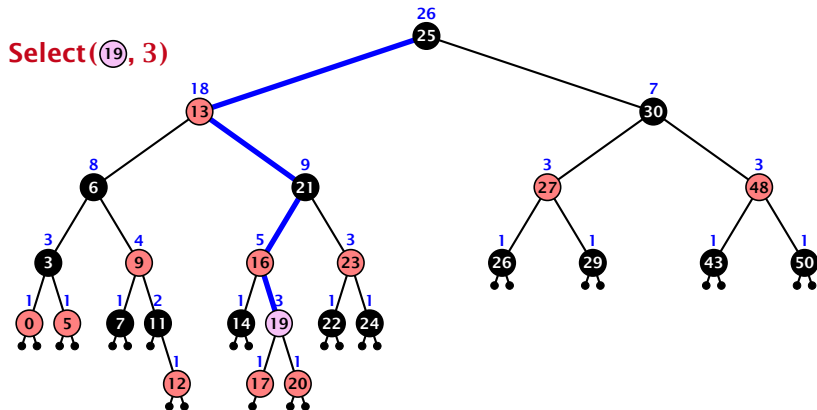
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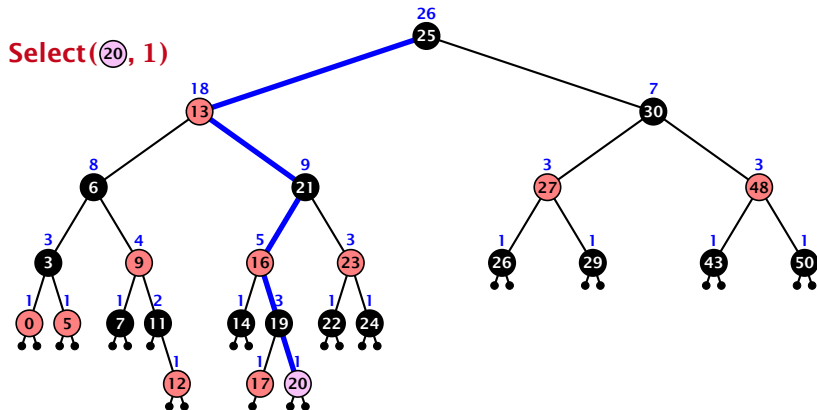
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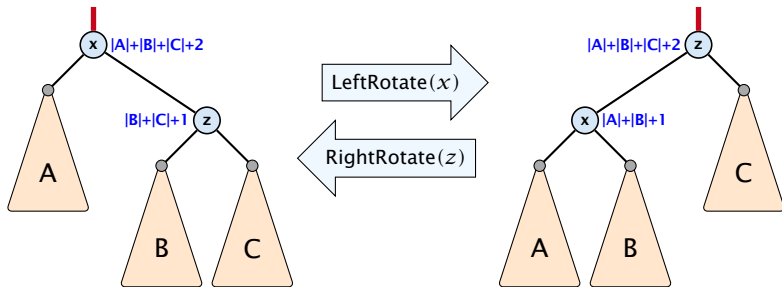
Search(k): Nothing to do.

Insert(x): When going down the search path increase the size field for each visited node. **Maintain the size field during rotations.**

Delete(x): Directly after splicing out a node traverse the path from the spliced out node upwards, and decrease the size counter on every node on this path. **Maintain the size field during rotations.**

Rotations

The only operation during the fix-up procedure that alters the tree and requires an update of the size-field:



The nodes x and z are the only nodes changing their size-fields.

The new size-fields can be computed **locally** from the size-fields of the children.