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Assignment 11
January 16, 2017

Efficient Algorithms and Data Structures I

Deadline: January 23, 2017, 10:15 am in the ***Efficient Algorithms*** *mailbox.*

Homework 1 (5 Points)

Let $G = (V, E)$ be a flow network with source s, sink t, and integer capacities. Let $n = |V|$ and $m = |E|$. Suppose that we are given a maximum flow in G.

(a) Suppose that the capacity of a single edge $(u, v) \in E$ is increased by 1. Give an $\mathcal{O}(m + n)$ time algorithm to update the maximum flow.

(b) Suppose that the capacity of a single edge $(u, v) \in E$ is decreased by 1. Give an $\mathcal{O}(m + n)$ time algorithm to update the maximum flow.

Homework 2 (5 Points)

Archibald Anderson must coordinate the traffic of football fans from Munich central to the football stadium. He studies the effect of road blocks on the ability to transport the fans. He models the situation using a flow network. His manager asks him to determine the *most vital edge*, whose deletion causes the larges decrease in the maximum flow value. Let $f : E \to \mathbb{N}$ be a maximum flow in the network. Either prove or disprove (using a counterexample) the following claims

1. A most vital edge is an edge with maximum capacity.
2. A most vital edge is an edge with maximum value of $f(e)$.
3. A most vital edge is an edge whose flow $f(e)$ equals the maximum value of $f(e')$ among edges e' belonging to some minimum cut.
4. An edge that does not belong to some minimum cut cannot be a most vital edge.
5. A network might contain several most vital edges.

Homework 3 (5 Points)

The edge connectivity of an undirected graph is the minimum number of edges that must be removed to disconnect the graph.

(a) Show that for any k, there exists a graph with edge connectivity k.

(b) Show how to determine the edge connectivity of an undirected graph G of n vertices by running a black-box maximum-flow algorithm on at most n flow networks.

Homework 4 (5 Points)

Consider a 0-1 matrix A with n rows and m columns. We refer to a row or a column of the matrix A as a *line*. We say that a set of 1's in the matrix A is *independent* if no two of them appear in the same line. We also say that a set of lines in the matrix is a *cover* of A if they cover all the 1's in the matrix. Using the max-flow min-cut theorem, show that the maximum number of independent 1's equals the minimum number of lines in a cover.

Tutorial Exercise 1

A town has r residents $R_1, \dots, R_r$, q clubs $C_1, \dots, C_q$, and p political parties $P_1, \dots, P_p$. Each resident is a member of at least one club and belongs to exactly one political party. Each club must nominate one of its members to represent it on the town's governing council so that the number of council members belonging to the political party P_k is at most u_k. Using maximum flows, find out whether it is possible for clubs nominate members in such a way.

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Everything changes and nothing re-
mains still ... and ... you cannot
step twice into the same stream.
    - Heraclitus (as quoted by Plato)
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