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Assignment 12
January 23, 2017

Efficient Algorithms and Data Structures I

Deadline: January 30, 2017, 10:15 am in the ***Efficient Algorithms*** *mailbox.*

Homework 1 (5 Points)

(a) We want to identify a minimum cut containing the least number of edges. Show how to rescale the edge capacities to find such a cut using a blackbox max-flow algorithm.

Hint: First assume that all capacities are at least 1.

(b) Consider the execution of the generic push-relabel algorithm. Show that, if all active nodes have label larger than n, then the value of the maximum flow equals the total flow entering the sink in the current preflow.

Homework 2 (3 Points)

Construct a family of networks with the number of minimum cuts growing exponentially with n, where n is the number of nodes in the network.

Homework 3 (6 Points)

The members of the *patisserie club* has n members $f_1, f_2, \ldots, f_n$, who have bought n cakes $c_1, c_2, \ldots, c_n$. Each club member wants to eat exactly one cake. Each club member chooses two cakes he would like to eat, and ranks them according to his preference. Rank 1 indicates higher preference, rank 2 indicates lower preference.

(a) We say that a cake assignment is a *feasible* assignment if every cake aficionado eats a cake within his (or her) preference list. How would you efficiently determine whether the club can find a feasible assignment?

(b) We say that a feasible assignment is an *optimal assignment* if it maximizes the number of club members assigned to their most preferred cake. Suggest an efficient algorithm for determining an optimal assignment and analyze its complexity.

Hint: Use a minimum cost flow.

Homework 4 (6 Points)

A path cover of a directed graph $G = (V, E)$ is a set P of vertex-disjoint paths such that every vertex in V is included in exactly one path in P. Paths may start and end anywhere, and they may be of any length, including 0. A minimum path cover of G is a path cover containing the fewest possible paths.

(a) Give an efficient algorithm to find a minimum path cover of a directed acyclic graph $G = (V, E)$.

Hint: Assuming that $V = \{1, 2, \ldots, n\}$, construct the graph $G' = (V', E')$,where

$$\begin{aligned} V' &= \{x_0, x_1, \ldots, x_n\} \cup \{y_0, y_1, \ldots, y_n\}, \\ E' &= \{(x_0, x_i) : i \in V\} \cup \{(y_i, y_0) : i \in V\} \cup \{(x_i, y_j) : (i, j) \in E\} \end{aligned}$$

and run a maximum-flow algorithm.

(b) Does your algorithm work for directed graphs that contain cycles? Explain or give a counterexample.

Tutorial Exercise 1

A shipping company wants to phase out a fleet of s (homogeneous) cargo ships over a period of p years. Its objective is to maximize its cash assets at the end of the p years by considering the possibility of prematurely selling ships and temporarily replacing them by charter ships.

The company faces a known nonincreasing demand for ships. Let d_k denote the demand of ships in year k. Each ship earns a revenue of r_k units in period k. At the beginning of year k, the company can sell any ship that it owns, accruing a cash inflow of s_k dollars. If the company does not own sufficiently many ships to meet its demand, it must hire additional charter ships. Let h_k denote the cost of hiring a ship for the kth year.

The shipping company wants to meet its commitments and at the same time maximize the cash assets at the end of the pth year.

Model this problem as a minimum cost flow problem!

`Network flow problems [...] arise in surprising ways in optimization problems that on the surface might note appear to involve networks at all.`

`- R.K. Ahuja, T.L. Magnanti, J.B. Orlin`