Technische Universität München
Fakultät für Informatik
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Winter term 2018/19
Assignment 3
October 29, 2017

Efficient Algorithms and Data Structures I

*Deadline: November 5, 10:15 am in the **Efficient Algorithms** mailbox.*

Homework 1 (5 Points)

(a) Give tight asymptotic upper and lower bounds for $T(n) = 2T(n/4) + \sqrt{n}\log_2 n$.

(b) For any constant $a \geq 27$ and n suitably large, let

$$T_a(n) = a \cdot T_a(n/4) + n^2 \quad .$$

Give all $a \geq 64$ such that $T_a(n) \in \Omega(n^4)$.

Homework 2 (5 Points)

Given two $n \times n$ matrices A and B where n is a power of 2, we know how to find $C = A \cdot B$ by performing n^3 multiplications. Now let us consider the following approach. We partition A, B and C into equally sized block matrices as follows:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \qquad C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Consider the following matrices:

$$\begin{aligned} M_1 &= (A_{11} + A_{22}) \cdot (B_{11} + B_{22}) \\ M_2 &= (A_{21} + A_{22}) \cdot B_{11} \\ M_3 &= A_{11} \cdot (B_{12} - B_{22}) \\ M_4 &= A_{22} \cdot (B_{21} - B_{11}) \\ M_5 &= (A_{11} + A_{12}) \cdot B_{22} \\ M_6 &= (A_{21} - A_{11}) \cdot (B_{11} + B_{12}) \\ M_7 &= (A_{12} - A_{22}) \cdot (B_{21} + B_{22}) \end{aligned}$$

Then,

$$C_{11} = M_1 + M_4 - M_5 + M_7$$

(a) Construct the matrices C_{12}, C_{21} and C_{22} from the matrices M_i, as demonstrated for C_{11}.

(b) Design an efficient algorithm for multiplying two $n \times n$ matrices based on these facts. Analyze its running time.

Homework 3 (5 Points)

Give tight asymptotic upper and lower bounds for $T(n)$, where $T(0)$ is an arbitrary constant, for the following recurrence relations

(a) $T(n) = T(n/2) + T(n/4) + T(n/8) + n$. for $n \geq 1$

(b) $T(n) = T(n/2 - 1) + 1$ for $n \geq 1$.

As argued in the lecture you may ignore the fact that function arguments can be non-integer.

Homework 4 (5 Points)

The recursion $T(n)$ is

$$T(n) = \sqrt{n}T(\sqrt{n}) + n \ .$$

Assuming that $T(n)$ is constant for sufficiently small n, show by induction that $T(n) = \Theta(n \log \log n)$.
Hint: You may assume that all logarithms in this exercise are binary.

Tutorial Exercise 1

The H-graph of order 0 is just a simple node. The H-graphs of order 1, 2, 3, and 4 are shown in Figure 1, Figure 2, Figure 3, and Figure 4, respectively. Let $f(\ell)$ denote the number of vertices of an H-graph of order ℓ. Develop a recurrence relation for f and solve your relation using techniques from the lecture.

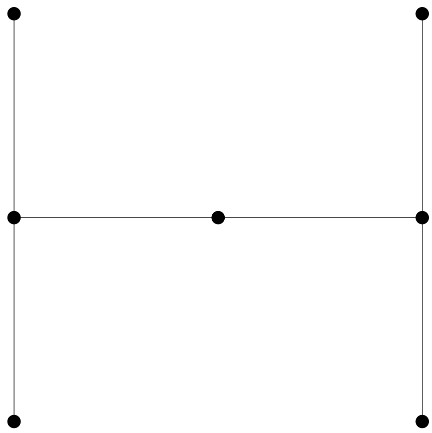

Figure 1: H-graph of order 1

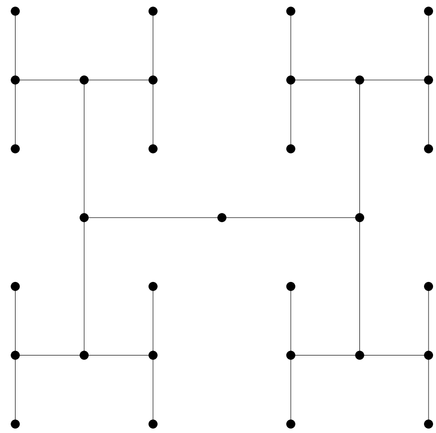

Figure 2: H-graph of order 2

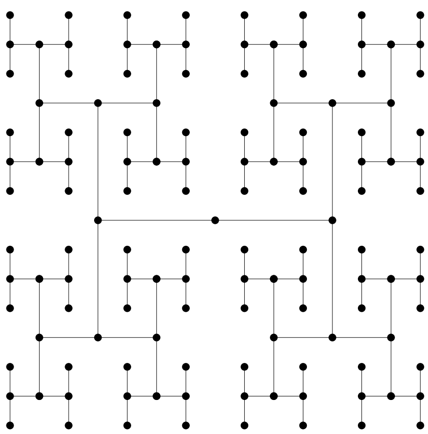

Figure 3: H-graph of order 3

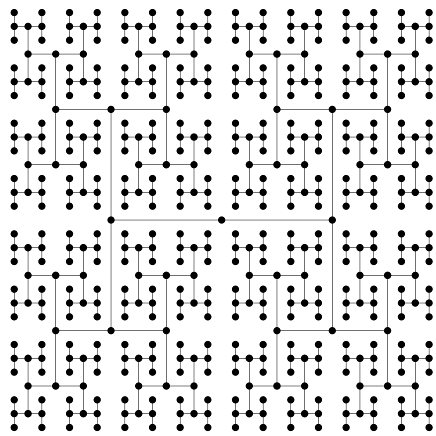

Figure 4: H-graph of order 4

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I like trees because they
seem more resigned to the
way they have to live than
other things do.
                - W. Cather
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