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Winter term 2019/20
Assignment 01
October 21, 2019

Efficient Algorithms and Data Structures I

Deadline: None, Tutorial exercises only.

Tutorial Exercise 1

For constants $c > 0$, $0 < \varepsilon < 1/2$ and $k > 1$, arrange the following functions of n in non-decreasing asymptotic order so that $f_i(n) \in O(f_{i+1}(n))$ for two consecutive functions in your sequence. Also indicate whether $f_i(n) \in \Theta(f_{i+1}(n))$ holds or not.

$$n^k, n^{1+sin(n)}, \log(n!), n^{k+\varepsilon}, n^n, n, n^k(\log n)^c, n!, 2^n, 3^n, n\log\log n, n\log(n), n^\varepsilon, n^{1/\log n} \ .$$

Here, log denotes the natural logarithm.

Tutorial Exercise 2

Let $f, g : \mathbb{N} \to \mathbb{R}^+$ be two positive monotonically increasing functions.
Prove or disprove the following statements. Use precise arguments based on the definition of the Landau-notation shown in the lecture.

1. For any positive, monotone increasing function $f : \mathbb{R} \to \mathbb{R}$, it holds that $f(\log_2(n)) \in \Theta(f(\log_4(n)))$.
2. $f(n) \in \Theta(f(n/4))$.

Tutorial Exercise 3

Jonathan is frustrated. His boss has given him a list of integers $a_1, \ldots, a_n$ with a weird assignment: For each $i \in [n]$, compute the product $b_i = \prod_{j\in[n], j\neq i} a_j$. Unfortunately, the latest update of his operating system has broken the division operator on his machine.

(a) At first, Jonathan's logarithm operator is still working well (in $\mathcal{O}(1)$ time). Find a way for him to compute all b_i's in $\mathcal{O}(n)$ steps.

(b) The next update to his system breaks the logarithm operator as well. Help Jonathan find a new $\mathcal{O}(n)$ time algorithm to compute all b_i's.

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The advanced reader who skips parts
that appear too elementary may miss
more than the less advanced reader
who skips parts that appear too com-
plex.
                          - G. Pòlya
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